

# Reshaping Cuboids

Junior high school

## Objective

Reinforce students' understanding of the equivalence of shapes in various alignments and how this relates to multiplication within a practical context.

Develop mental skills involving factors, divisors, and systematic thinking.

## Explanation of the activity

12 cubes, each with a volume of  $1 \text{ cm}^3$ , may be placed together to create any of four cuboids, each having a volume of  $12 \text{ cm}^3$ .

Find the equivalent equations for each of the cuboids; for example,

$1 \times 1 \times 12 = 12$ ,  $2 \times 2 \times 3 = 12$ , etc.

## Using the calculator

Calculator functions used: Multiplication, Multi-line Playback

Press the following buttons and then start operation.

**ON/C** **MODE** **0**

Introduce students to the calculator's Multi-line playback feature, which will be useful to display sets of solutions for each "volume" number.

1 **×** 1 **×** 12 **=**

1X1X12= <sup>DEG</sup> 12.

2 **×** 2 **×** 3 **=**

2X2X3= <sup>DEG</sup> 12.

1 **×** 3 **×** 4 **=** etc.

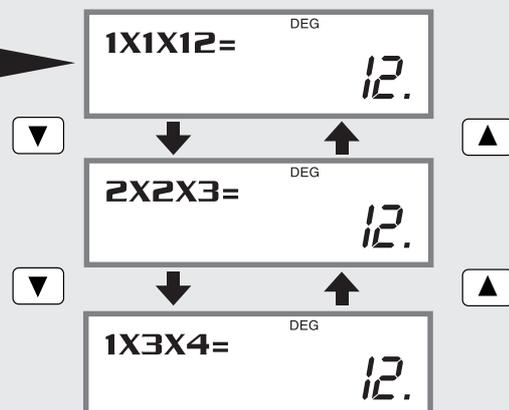
1X3X4= <sup>DEG</sup> 12.

Each press of a **▲** **▼** key takes you one calculation step forward or backward.

Display the first calculation

with **2ndF** **▲**

**2ndF** **▲**



Find the five calculations that represent cuboids that each have a volume of  $30 \text{ cm}^3$ .

e.g.  $1 \times 1 \times 30 =$  etc.

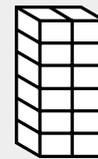
In a similar way, find the twelve calculations for cuboids each having a volume of  $96 \text{ cm}^3$ .

How many similar calculations must there be for  $180 \text{ cm}^3$ ?  
Which of these cuboids is nearest to looking like a cube?

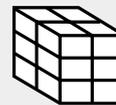
For volumes between  $150 \text{ cm}^3$  and  $200 \text{ cm}^3$ , which particular ones can be represented by at least 16 cuboids each? Which volumes have the smallest number of cuboids each?



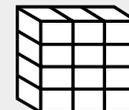
$$1 \times 1 \times 12 = 12$$



$$1 \times 2 \times 6 = 12$$



$$2 \times 2 \times 3 = 12$$



$$1 \times 3 \times 4 = 12$$

## ••••• Using the activity in the classroom •••••

Students may benefit from the use of actual blocks that can be stacked to form the different cubic combinations. An OHP calculator could also be used to collect solutions from the entire class.

## ••••• Points for students to discuss •••••

The number of divisors for a number expressed as  $p^a \times q^b \times r^c$  (where  $p, q,$  and  $r$  are all prime) is  $(a + 1)(b + 1)(c + 1)$ . For example,  $360 = 2^3 \times 3^2 \times 5^1$ . Here,  $a = 3, b = 2,$  and  $c = 1$ , so the number of divisors is given by the expression  $(3 + 1)(2 + 1)(1 + 1) = 24$ . Therefore, 360 has 24 divisors.

### Further Ideas

- Use trial and improvement to find the side of a cube having a volume of  $180 \text{ cm}^3$ .
- Move into "four (or more) dimensions" as a means of finding the factors of a number. For example,  $6006 = 77 \times 78 = (7 \times 11) \times (6 \times 13) = 2 \times 3 \times 7 \times 11 \times 13$ . All stages can be displayed using the replay function.