Evaluating Absolute Value Functions

The absolute value of a real number x is defined by the following:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

Note that the effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative.
Thus, $|x| \geq 0$ for all values of x.

Example

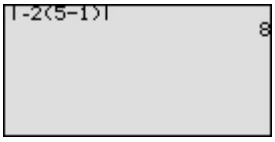
Evaluate various absolute value functions.

1. Evaluate $|-2(5-1)|$

2. Is $|-2+7| = |-2| + |7|$?
Evaluate each side of the equation to check your answer.
Is $|x + y| = |x| + |y|$ for all real numbers x and y?
If not, when will $|x + y| = |x| + |y|$?

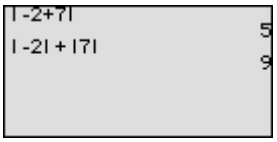
3. Is $\left|\frac{6-9}{1+3}\right| = \frac{|6-9|}{|1+3|}$?
Evaluate each side of the equation to check your answer. Investigate with more examples, and decide if you think $|x / y| = |x| / |y|$

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Step & Key Operation	***Display***	***Notes***
1-1 Access the home or computation screen. [⊞]		
1-2 Enter $y = |-2(5-1)|$ and evaluate. [MATH] [B] [1] [(-)] [2] [(] [5] [—] [1] [)] [ENTER]		The solution is ±8.

2-1 Evaluate $|-2 + 7|$. Evaluate $|-2| + |7|$.

[CL]
[MATH] [1] [(-)] [2] [+] [7] [ENTER]
[MATH] [1] [(-)] [2] [▶] [+] [MATH] [1] [7] [ENTER]



$|-2 + 7| = 5$, $|-2| + |7| = 9$
$\rightarrow |-2 + 7| \neq |-2| + |7|$.

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Step & Key Operation | ***Display*** | ***Notes***

2-2

Is $|x + y| = |x| + |y|$? Think about this problem according to the cases when x or y are positive or negative.

If $x \geq 0$ and $y \geq 0$
[e.g.; $(x, y) = (2,7)$]

$|x + y| = |2 + 7| = 9$
$|x| + |y| = |2| + |7| = 9$
$\rightarrow |x + y| = |x| + |y|$.

If $x \leq 0$ and $y \geq 0$
[e.g.; $(x, y) = (-2, 7)$]

$|x + y| = |-2 + 7| = 5$
$|x| + |y| = |-2| + |7| = 9$
$\rightarrow |x + y| \neq |x| + |y|$.

If $x \geq 0$ and $y \leq 0$
[e.g.; $(x, y) = (2, -7)$]

$|x + y| = |2-7| = 5$
$|x| + |y| = |2| + |-7| = 9$
$\rightarrow |x + y| \neq |x| + |y|$.

If $x \leq 0$ and $y \leq 0$
[e.g.; $(x, y) = (-2, -7)$]

$|x + y| = |-2-7| = 9$
$|x| + |y| = |-2| + |-7| = 9$
$\rightarrow |x + y| = |x| + |y|$.

Therefore $|x + y| = |x| + |y|$ when $x \geq 0$ and $y \geq 0$, and when $x \leq 0$ and $y \leq 0$.

3-1

Evaluate $\left|\frac{6-9}{1+3}\right|$. Evaluate $\frac{|6-9|}{|1+3|}$.

$\left|\frac{6-9}{1+3}\right| = 0.75$, $\frac{|6-9|}{|1+3|} = 0.75$
$\rightarrow \left|\frac{6-9}{1+3}\right| = \frac{|6-9|}{|1+3|}$

CL MATH 1 a/b 6 — 9

▶ 1 + 3 ENTER

MATH 1 6 — 9 ▶ a/b

MATH 1 1 + 3 ENTER

3-2

Is $|x / y| = |x| / |y|$?
Think about this problem according to the cases when x or y are positive or negative.

If $x \geq 0$ and $y \geq 0$
[e.g.; $(x, y) = (2,7)$]

$|x / y| = |2/7| = 2/7$
$|x| / |y| = |2| / |7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

If $x \leq 0$ and $y \geq 0$
[e.g.; $(x, y) = (-2, 7)$]

$|x / y| = |(-2)/7| = 2/7$
$|x| / |y| = |-2| / |7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

If $x \geq 0$ and $y \leq 0$
[e.g.; $(x, y) = (2, -7)$]

$|x / y| = |2/(-7)| = 2/7$
$|x| / |y| = |2| / |-7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

If $x \leq 0$ and $y \leq 0$
[e.g.; $(x, y) = (-2, -7)$]

$|x / y| = |(-2)/-7| = 2/7$
$|x| / |y| = |-2| / |-7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

The statement is true for all $y \neq 0$.

The EL-9900 shows absolute values with | |, just as written on paper, by using the Equation editor. The nature of arithmetic of the absolute value can be learned through arithmetical operations of absolute value functions.