APPLYING

TRIGONOMETRY

SHARP

USING THE

EL-9600

Graphing Calculator

DAVID P. LAWRENCE

Applying
TRIGONOMETRY
using the
SHARP EL-9600
GRAPHING CALCULATOR

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This Teaching Resource has been developed specifically for use with the Sharp EL-9600 graphing calculator. The goal for preparing this book was to provide mathematics educators with quality teaching materials that utilize the unique features of the Sharp graphing calculator.

This book, along with the Sharp graphing calculator, offers you and your students 10 classroom-tested, topic-specific lessons that build skills. Each lesson includes *Introducing the Topic*, *Calculator Operations*, *Method of Teaching*, explanations for *Using Blackline Masters*, *For Discussion*, and *Additional Problems* to solve. Conveniently located in the back of the book are 36 reproducible *Blackline Masters*. You'll find them ideal for creating handouts, overhead transparencies, or to use as student activity worksheets for extra practice. *Solutions to the Activities* are also included.

We hope you enjoy using this resource book and the Sharp El-9600 graphing calculator in your classroom.

Other books are also available:

Applying STATISTICS using the SHARP EL-9600 Graphing Calculator
Applying PRE-ALGEBRA and ALGEBRA using the SHARP EL-9600 Graphing Calculator
Applying PRE-CALCULUS and CALCULUS using the SHARP EL-9600 Graphing Calculator
Graphing Calculators: Quick & Easy! The SHARP EL-9600

Dedicated to my grandma, Marjorie Lee

Special thanks to Ms. Marina Ramirez and Ms. Melanie Drozdowski for their comments and suggestions.

Developed and prepared by Pencil Point Studio.

CONTENTS

Chapter one

# GRAPHS OF TRIGONOMETRIC FUNCTIONS

Introducing the Topic

In this chapter, you and your students will learn how to graph trigonometric functions on the Sharp EL-9600. Graphing trigonometric functions on the calculator is covered first, so that graphical evaluation of trigonometric functions may be covered in the next chapter. The trigonometric functions are often introduced as circular functions with angle measurement in degrees and radians. The sine and cosine functions are said to map the angle measurement to the y and x coordinates respectively, of the point on the unit circle representing the angle. Then, the four additional trigonometric functions are defined in terms of sine and cosine.

If you let the angle measurement be represented by a horizontal axis (x) and let the vertical axis (y) represent the function evaluated at the angle, then you can graph the six trigonometric functions as $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, and $y = \cot x$.

Calculator Operations

Turn the calculator on and press $\boxed{\text{Y=}}$. The Y prompts will appear in the viewing window. Press $\boxed{\text{CL}}$ to remove an old Y1 expression. Press $\boxed{\blacktriangledown}$ $\boxed{\text{CL}}$ to clear additional expressions. The calculator should be setup in radian measurement, with rectangular coordinates, and in equation editor mode. To complete this setup, press $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, double touch **2 Rad**, touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ again to exit the menu and return to the Y prompts.

To enter the sine function ($y = \sin x$) for Y1, press $\boxed{\text{sin}}$ $\boxed{\text{X/}\theta\text{/T/}n}$. Enter the viewing window range by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, and double touching **1 sin x**. This establishes built-in viewing-window range for graphing the sine function. The window ranges are $^-2\pi \leq x \leq 2\pi$ and $^-1.55 \leq y \leq 1.55$. The scale for the x axis is $\frac{\pi}{2}$ and the y-axis scale is .5.
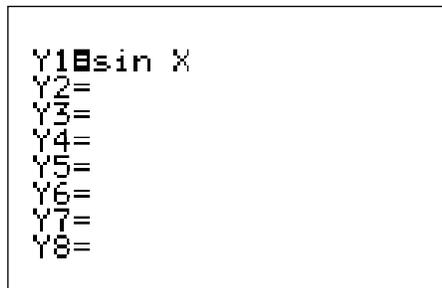
```
Y1█sin X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
```
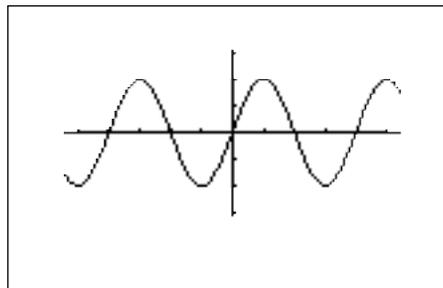
The remaining five trigonometric functions can be graphed similarly.

Method of Teaching

Use the *Blackline Master 1.1* to create an overhead for graphing the sine function. Go over in detail how to graph a trigonometric function. Follow the sine function demonstration with the cosecant function demonstration provided in the sample problem below and on the Blackline Master.

Next, use the *Blackline Masters 1.2* through *1.4* to create worksheets for the students. Have the students graph the remaining four trigonometric functions. Use the topics *For Discussion* to supplement the worksheets. Engage the trace feature by pressing TRACE . Move the tracer by pressing ◄ or ► . Tracing will assist you in discussing the topics.

Using Blackline Master 1.1

The problem discussed above in the *Calculator Operations* section is presented on *Blackline Master 1.1*. The following problem also appears on the Blackline Master.

Graph the cosecant function by pressing Y= CL to remove an old Y1 expression. To enter the cosecant function for Y1, press MATH , you may need to touch **A CALC** if the menu is not already there. Double touch **10 csc**, and press X/θ/T/*n* . Use the sine function's built-in viewing range, and press GRAPH to view the graph.

```
Y1█csc X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
```
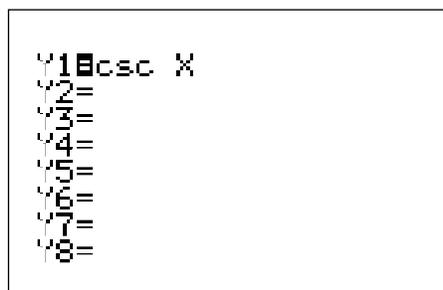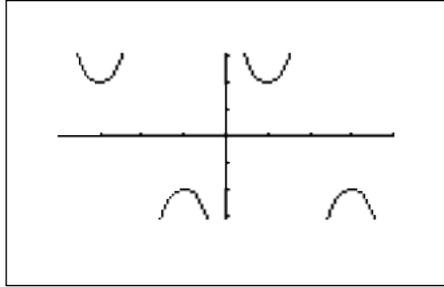
For Discussion

You and your students can use the calculator to discuss:

(1) intervals where the functions increase and decrease,

(2) domain and range of the functions,

(3) the periodicity of the functions, and

(4) which part of the horizontal axis represents each quadrant.

The trace feature will allow you to move along the curve to see where the function is increasing and decreasing, to see the values associated with limits on the domain and range, to see the period of a function, and to explore the quadrant "markers" on the *x*-axis.

Chapter two

EVALUATING TRIGONOMETRIC FUNCTIONS

Introducing the Topic

In this chapter, you and your students will learn how to evaluate trigonometric functions directly in radians or degrees, and graphically through the tracing feature.

Calculator Operations

To evaluate sin 45° directly, you will turn the calculator on and press $\boxed{\begin{smallmatrix}+ & - \\ \times & \div\end{smallmatrix}}$ to enter computation mode. Press $\boxed{\text{CL}}$ to clear the viewing window of any old expressions. To evaluate trigonometric functions in degrees, you can set the calculator to operate in degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu. Now, to find the sin 45° you will press $\boxed{\text{sin}}$ $\boxed{4}$ $\boxed{5}$ $\boxed{\text{ENTER}}$.
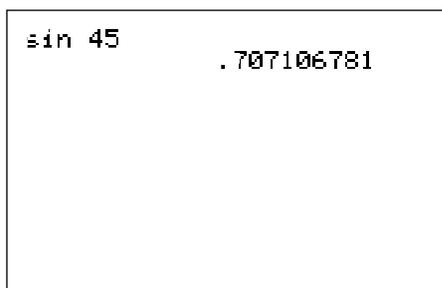
```
sin 45
            .707106781
```

To approximate sin 45° graphically, you will press $\boxed{\text{Y=}}$ followed by $\boxed{\text{CL}}$ to clear an old expression for Y1. Enter the function sin *x* by pressing $\boxed{\text{sin}}$ $\boxed{\text{X/θ/T/}n}$. To view the graph press $\boxed{\text{ZOOM}}$, touch **E TRIG**, and double touch **1 sin *x***. Now, engage the trace by pressing $\boxed{\text{TRACE}}$ and moving the tracer near *x* = 45° by pressing $\boxed{\blacktriangleright}$.
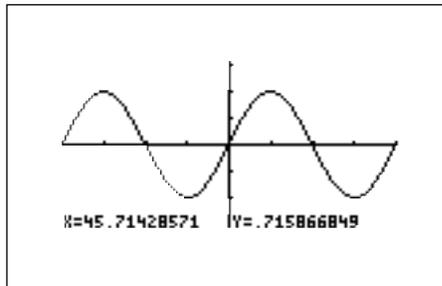
With the cursor located near *x* = 45°, you can zoom in to find a better approximate for the *y* value. First, set the factors for zooming by pressing $\boxed{\text{ZOOM}}$, touching **B FACTOR**, and pressing $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{\text{ENTER}}$. These larger values will accelerate the zooming in. Now, press $\boxed{\text{ZOOM}}$, touch **A ZOOM**, and double touch **3 In** to zoom in on the cursor. Once again, trace over near *x* = 45°. Zoom in two more times to see a better approximation.
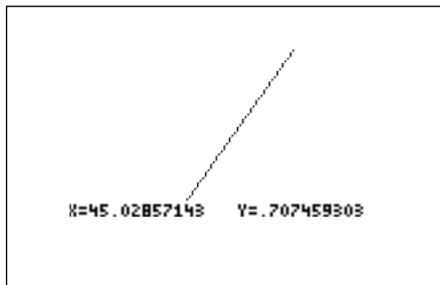
To find sin $\frac{\pi}{3}$ directly, you will turn the calculator on and press $\boxed{\boxminus}$ to enter computation mode. Press $\boxed{\text{CL}}$ to clear the viewing window of any old expressions. To evaluate trigonometric functions in radian mode, you can set the calculator to operate in radians by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu. Now, to find sin $\frac{\pi}{3}$ you will press $\boxed{\text{sin}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{a/b}}$ $\boxed{3}$ $\boxed{\text{ENTER}}$.

$$\sin \frac{\pi}{3}$$

.866025403

To approximate $\sin \frac{\pi}{3}$ graphically, you will press $\boxed{Y=}$ followed by $\boxed{CL}$ to clear an old expression for Y1. Enter the function $\sin x$ by pressing $\boxed{\sin}$ $\boxed{X/\theta/T/n}$. View the graph by pressing $\boxed{ZOOM}$, touching **E TRIG**, double touching **1 sin x**. Now, engage the trace by pressing $\boxed{TRACE}$ and tracing near $x = \frac{\pi}{3} = 1.047$.
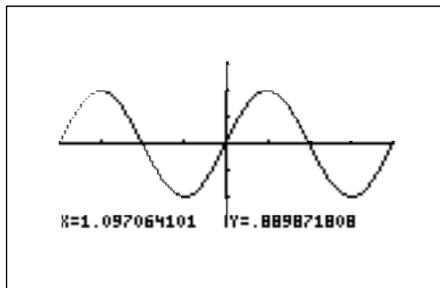
X=1.097064101   Y=.889871808

With the cursor located near $x = 1.047$, you can zoom in to find a better approximation for the y value. Press $\boxed{ZOOM}$, touch **A ZOOM**, and double touch **3 In** to zoom in on the cursor. Once again, trace over near $x = 1.047$. Zoom in one more time to see an even better approximation.

X=1.049192213   Y=.867021011
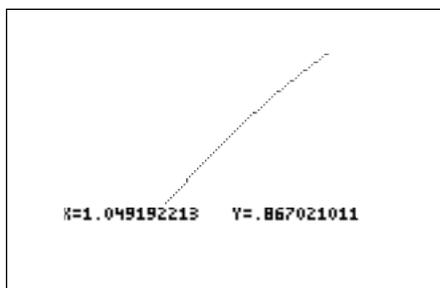
Method of Teaching

Use *Blackline Master 2.1* to create an overhead or handout for direct and graphical evaluation of trigonometric functions. Demonstrate the procedure changes for radian measurement. Remind the students of the location of cosecant, secant, and cotangent within the MATH menu.

Next, use the *Blackline Masters 2.2* through *2.4* to create worksheets for the students. Have the students evaluate the six trigonometric functions directly and graphically for the various degree and radian measurements. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 2.2

The following problem appears on *Blackline Master 2.2*. Evaluate sec $\frac{\pi}{4}$ directly in degree mode. Press $\boxed{\boxminus}$ $\boxed{CL}$ to enter computation mode and clear the home screen. Press $\boxed{2ndF}$ $\boxed{SET\ UP}$ to verify the calculator is in degree mode. If it is not, change it to degree mode. Press $\boxed{2ndF}$ $\boxed{QUIT}$ to exit the menu. Enter and evaluate the expression by pressing $\boxed{MATH}$, touching **A CALC**, double touching **09 sec**, pressing $\boxed{2ndF}$ $\boxed{\pi}$ $\boxed{a/b}$ $\boxed{4}$ $\boxed{\blacktriangleright}$ $\boxed{MATH}$, touching **E ANGLE**, double touching **4 r**, and pressing $\boxed{ENTER}$.
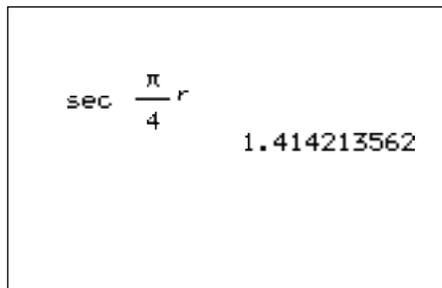
```
sec  π  r
     ─
     4
             1.414213562
```

Evaluate tan 20° graphically. Press $\boxed{Y=}$ and press $\boxed{CL}$ to remove an old expression for Y1. Press $\boxed{2ndF}$ $\boxed{SET\ UP}$ to verify the calculator is in degree mode. If it is not, change it. Press $\boxed{2ndF}$ $\boxed{QUIT}$ to exit the menu. Enter the function tan x for Y1 by pressing $\boxed{tan}$ $\boxed{X/\theta/T/n}$. Instead of graphing the function in the default viewing window for tangent, press $\boxed{WINDOW}$ adjust the Xmin and Xmax to 10° and 30° (value of interest ± 10°) respectively to start out closer to the value of interest. Set the Xscl = 1. Press $\boxed{ZOOM}$, touch **A ZOOM**,

and double touch **1 Auto** to view the graph. Now, engage the trace by pressing $\boxed{\text{TRACE}}$. Notice the tracer appears at $x = 20°$ since it is the middle of the x values.

X=20   Y=0.363970234
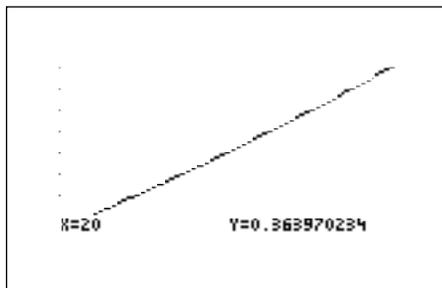
For Discussion

Students can calculate sin 25° in radian mode by pressing $\boxed{\text{sin}}$ $\boxed{2}$ $\boxed{5}$ $\boxed{\text{MATH}}$, touching **E ANGLE**, double touching **1°**, and pressing $\boxed{\text{ENTER}}$. Students can store a graph with a particular viewing window for repeated use by using the StoGD command in the DRAW menu. For more detail, refer to the EL-9600 Owner's Manual. Discuss the use of the negative sign like cos (⁻10°). This is the first assignment that uses a negative angle. Also remind them of the difference between the negative key (a short dash within parentheses) and the minus key (no parentheses).

Additional Problems

Evaluate the following directly:

(1) cos 27°

(2) cot 52°

(3) sec $\dfrac{5\pi}{3}$

Evaluate the following graphically:

(4) sin 107°

(5) tan $\dfrac{\pi}{7}$

(6) csc $\dfrac{5\pi}{4}$

Chapter three

GRAPHICAL VERIFICATION OF FORMULAS AND IDENTITIES

Introducing the Topic

In this chapter, you and your students will learn how to verify trigonometric formulas and identities graphically. The left and right sides of an equality can be graphed by the calculator as Y1 and Y2. Verification is made by viewing Y1 and Y2 as the same graph, hence equivalent expressions.

Calculator Operations

To verify the double angle formula $\sin 2\theta = 2 \sin \theta \cos \theta$, you will graph the "sin 2θ" as Y1 = sin ($2x$) and the "2 sin θ cos θ" as Y2 = 2 sin x cos x and show that they are in fact the same graph. To do this you will turn the calculator on and press $\boxed{\text{Y=}}$. Press $\boxed{\text{CL}}$ to clear Y1 of an old expression. Press $\boxed{\blacktriangledown}$ $\boxed{\text{CL}}$ to clear additional expressions. Make sure the calculator is in radian mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu.

Now enter sin ($2x$) for Y1 by pressing $\boxed{\text{sin}}$ $\boxed{(}$ $\boxed{2}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{)}$ $\boxed{\text{ENTER}}$ and the 2 sin x cos x for Y2 by pressing $\boxed{2}$ $\boxed{\text{sin}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{cos}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{ENTER}}$. Set your viewing window by pressing $\boxed{\text{WINDOW}}$ $\boxed{\text{EZ}}$, touch **5**, touch ❤ on the screen, touch **6** $^-2\pi$**<X,<2**π, touch ❤ on the screen, and touch **5** $^-$**5<Y<5**.
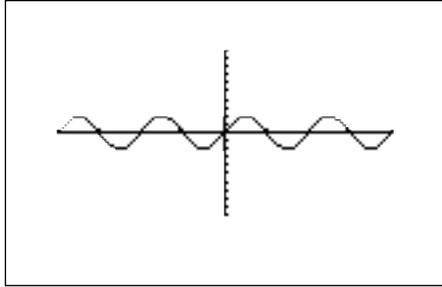
To verify the identity $\sin^2\theta + \cos^2\theta = 1$, you will graph the "$\sin^2\theta + \cos^2\theta$" as Y1 = $\sin^2 x + \cos^2 x$ and the "1" as Y2 = 1, and show that they are the same graph. To do this press $\boxed{Y=}$ to access the Y1 prompt. Press $\boxed{CL}$ to clear Y1 of an old expression, and enter the $\sin^2 x + \cos^2 x$ for Y1 by pressing $\boxed{(}$ $\boxed{\sin}$ $\boxed{X/\theta/T/n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{+}$ $\boxed{(}$ $\boxed{\cos}$ $\boxed{X/\theta/T/n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{2}$ $\boxed{ENTER}$. Enter the 1 for Y2 by pressing $\boxed{CL}$ $\boxed{1}$ $\boxed{ENTER}$. Press $\boxed{GRAPH}$ to view the graph.
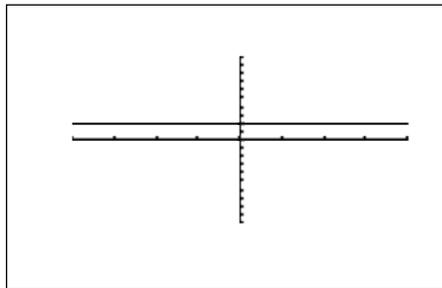
Method of Teaching

Use *Blackline Master 3.1* to create an overhead or handout for graphically verifying trigonometric formulas and identities. Demonstrate the process of entering the equality's left- and right-hand expressions and when necessary, emphasize the intervals where functions are positive or negative (as demonstrated in *Using Blackline Master 3.2*). Next, use the *Blackline Masters 3.3* through *3.5* to create worksheets for the students. Have the students verify the various formulas and identities. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 3.2

The following problem appears on *Blackline Master 3.2*. Verify the half angle formula $\cos \frac{\theta}{2} = \pm\sqrt{(1 + \cos \frac{\theta}{2})}$. To verify the formula, you will graph the "$\cos \frac{\theta}{2}$" as Y1 $= \cos \frac{x}{2}$ and the "$\pm\sqrt{\frac{1 + \cos \theta}{2}}$" as Y2 $= \pm\sqrt{\frac{1 + \cos \theta}{2}}$ and show that they are in fact the same graph. In this problem, you will need to limit your domain or *x* values to intervals where cosine is either negative or positive and enter the appropriate radical for Y2.

To do this, you will turn the calculator on and press Y= . Press CL to clear Y1 of an old expression.

Now enter the $\cos \frac{x}{2}$ for Y1 by pressing cos X/θ/T/*n* a/b 2 ENTER and the $\frac{1 + \cos x}{2}$ for Y2 (limiting to where cosine is positive) by pressing CL 2ndF √ a/b 1 + cos X/θ/T/*n* ▼ 2 ENTER . Cos $\frac{x}{2}$ will be positive for a viewing window of $0 < x < \pi$. Enter this viewing window by pressing WINDOW 0 ENTER 2ndF π ENTER 2ndF π ÷ 2 ENTER (–) 3 ENTER 3 ENTER 1 ENTER . Press GRAPH to view the graphs. Notice they appear as one graph.

To verify the formula for where cosine is negative, press Y= ▼ to access the Y2 prompt and press (–) to insert a negative in front of the radical. Cos $\frac{x}{2}$ will be negative for a viewing window of $\pi < x < 3\pi$. Enter this viewing window by pressing WINDOW 2ndF π ENTER 3 2ndF π ENTER .
Press GRAPH to view the graphs. Notice they appear as one graph.

For Discussion

Students can use the calculators to discuss how "toggling" (using the ▲ and ▼ keys while tracing) between Y1 and Y2 verifies the same values for both Y1 and Y2, and they can explore what happens when they do not limit the domain on those problems with ± parts (they see two graphs).

Additional Problems

Verify the following formulas and identities graphically.

1. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

2. $1 + \cot^2 \theta = \csc^2 \theta$

3. $\sin^2 \theta \ \cot^2 \theta = (1 + \sin \theta)(1 - \sin \theta)$

Chapter four
_____

AMPLITUDE, PERIOD, AND PHASE SHIFT

Introduction

In this chapter, you and your students will learn how to modify trigonometric functions by constants. If you multiply sine or cosine by a constant A, then every value of the function is multiplied by A. The maximum absolute value of the sine or cosine function is 1. Therefore, Asin θ and Acos θ will have a maximum value of |A|. The value of |A| is called the amplitude of the functions y = Asin x and y = Acos x, and is the distance sine and cosine rises and falls above the x-axis. Multiplication of the function by A will stretch or shrink the curve vertically. |A| > 1 will stretch the curve while an |A| between 0 and 1 will shrink the curve. The tangent function does not have an amplitude because tangent increases without bound as θ approaches values such as $\frac{\pi}{2}$.

If we multiply the argument θ by a constant k within a trigonometric function, then every measure of an angle θ is multiplied by k. Multiplication of θ will stretch or shrink the curve horizontally. A value of k > 1 will shrink the curve while a value of k between 0 and 1 will stretch the curve. The stretching and shrinking of a curve horizontally effects the function's period. The new periods will be $\frac{2\pi}{k}$ for sine, cosine, secant and cosecant, and $\frac{\pi}{k}$ for tangent and cotangent.

If we add a constant c to the argument θ within a trigonometric function, then every measure of an angle θ is increased by c. Addition of c will shift the function left or right. A $c > 0$ will shift the function to the left while a $c < 0$ will shift the function to the right.

If we combine both the multiplication by a constant k and the addition of a constant c to the argument θ (sin (kθ + c)), then the movement of the function left or right with respect to the period change is called the phase shift. The value for this shift is $\frac{-c}{k}$. A $c > 0$ will shift the function $\frac{c}{k}$ units to the left while a $c < 0$ will shift the function $\frac{c}{k}$ units to the right.

Calculator Operations

To investigate changes in amplitude, you will use the SHIFT/CHANGE feature of the graphing calculator. Press 2ndF SHIFT/CHANGE , touch **B CHANGE**, and double touch **5 Y=sin X**. Double touch the screen at **Y = 3**, above the box with the arrows. Notice the new function Y = 3 sin X is displayed. Press ENTER to place the new function in the list.

Double touch the screen at Y = ‾2, below the box with the arrows. Notice the new function Y = ‾2 sin X is displayed. Press ENTER to place the new function in the list.
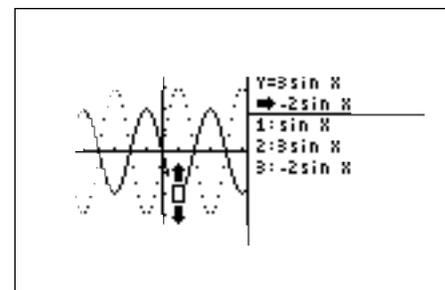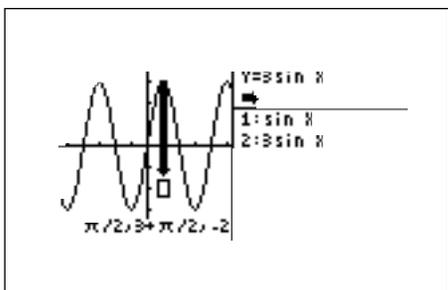
Touch the top of the right side of the screen and the list of three functions will be displayed. The function Y = sin X is highlighted on the right side of the screen, and the graph is darkened on the left side of the screen. Press ▼ to move from one graph to the next. Observe and note the changes in sin X when you change the amplitude.

Method of Teaching

Use *Blackline Master 4.1* to create an overhead or handout for demonstrating changes in amplitude. Next use *Blackline Masters 4.2* and *4.3* to create worksheets to demonstrate changes in period and phase shift. For analysis of period, demonstrate the process of entering a "standard" trigonometric function for Y1 and a changed function for Y2. Use the trace feature to verify the new period.

For analysis of phase shift, demonstrate the use of SHIFT feature. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Masters 4.2 and 4.3

The following problems appear on Blackline Masters 4.2 and 4.3. To investigate the change in a trigonometric function by multiplying the argument θ by 2, press Y= CL to access and clear Y1 of an old expression. Make sure the calculator is in radian mode by pressing 2ndF SET UP , touch **B DRG**, and double touch **2 Rad**. Press 2ndF QUIT to exit the menu.

Now, enter sin *x* for Y1 by pressing sin X/θ/T/*n* ENTER and the sin (2*x*) for Y2 by pressing CL sin (2 X/θ/T/*n*) ENTER . Set your viewing window by pressing ZOOM , touching **E TRIG**, and double touching **1 sin *x***. Note the change in the second graph versus the first graph. Engage the trace to show the period has changed from 2π to π.

To investigate the change in a trigonometric function by subtracting $\frac{3\pi}{2}$ to the argument θ, press ⌐2ndF⌐ ⌐SHIFT/CHANGE⌐ , touch **A SHIFT**, and double touch **6 Y=sin X**. Double touch the screen at **X = 3π/2** to the right of the box with the arrows. Notice the new function Y = sin (X - 3π/2) is displayed. Press ⌐ENTER⌐ to place the new function in the list.

Double touch the screen at **Y = ⁻2**, below the box with the arrows. Notice the new function Y = sin (X - 3π/2) – 2 is displayed. Press ⌐ENTER⌐ to place the new function in the list. Touch the top of the right side of the screen and the list of three functions will be displayed. The function Y = sin X is highlighted on the right side of the screen, and the graph is darkened on the left side of the screen. Press ⌐▼⌐ to move from one graph to the next. Observe and note the changes in sin X when you change the phase shift.

For Discussion

Students can use the calculators to discuss why tangent does not have an amplitude, the period and period changes of tangent, the phase shifts for tangent, and what happens when you multiply tangent by a constant.

Additional Problems

Use the following to demonstrate the period, period changes and phase shifts for tangent.

(1) Compare $\tan \theta$ with $(^-2)\tan \theta$

(2) Compare $\tan \theta$ with $\tan \frac{\theta}{3}$

(3) Compare $\tan \theta$ with $\tan (\theta - \frac{\pi}{2})$

(4) Compare $\tan \theta$ with $\tan (\theta + \frac{\pi}{2}) + 2$

Chapter five

INVERSE TRIGONOMETRIC FUNCTIONS

Introducing the Topic

In this chapter, you and your students will learn how to compute and graph inverse trigonometric functions. The inverse is found by interchanging the range and domain of the trigonometric function. Thus, the domain of the trigonometric function becomes the range of the inverse and the range of the trigonometric function becomes the domain of the inverse.

For example, the sine function has $\mathbb{R}$ as its domain and $^-1 \leq y \leq 1$ as its range. Therefore, the inverse of the sine function (arcsine or inverse sine) will have $^-1 \leq x \leq 1$ as its domain and $\mathbb{R}$ as its range. Notice, like some algebraic functions, the inverse of trigonometric function will not be a function. This is due to periodicity of the trigonometric function's domain extends to the inverse's range. Thus, one value in the inverse's domain will correspond to more than one value in the inverse's range. To make the inverse a function, you will limit the domain of the trigonometric function so it forms a one-to-one function.

Limit the domain of the sine and tangent functions to $^-\pi/2 \leq x \leq \pi/2$, and limit the cosine function to $0 \leq x \leq \pi$. These limited domains result in one-to-one functions whose inverses are functions. The values within these limited domains are called the principal values. Notations for "inverse sine" are "arcsin" and "sin^{-1} ".

When asked to mentally find arcsin (1), you will typically ask yourself the question: "Sine of what angle (within the principal values) will give me 1?" The calculator does this computation very quickly for you.

When graphing an inverse function by hand, you would swap the limited domain and the complete range values of the trigonometric function to form the inverse function. Once again, the calculator does this swap quickly for you.

Calculator Operations

To compute inverse trigonometric functions you will turn the calculator on and press $\boxed{\begin{smallmatrix}+ & - \\ \times & \div\end{smallmatrix}}$ $\boxed{\text{CL}}$ to enter computation mode and clear the home screen. The computations can be done in degrees and radians.

Let's do the first one in degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu. To find sin $^{-1}$ (.5), press $\boxed{\text{2ndF}}$ $\boxed{\text{sin}^{-1}}$ $\boxed{•}$ $\boxed{5}$ $\boxed{\text{ENTER}}$. The answer of 30° is shown below.

```
sin⁻¹ .5
                    30
```
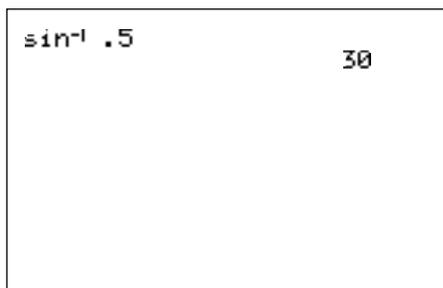
To graph the inverse sine function, press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear the Y1 prompt. Press $\boxed{\text{ENTER}}$ $\boxed{\text{CL}}$ to clear additional Y prompts. Enter sin⁻¹ x for Y1 by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{sin}^{-1}}$ $\boxed{\text{X/θ/T/}n}$ $\boxed{\text{ENTER}}$. Set your viewing window by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, and double touching **4 sin⁻¹ x**.
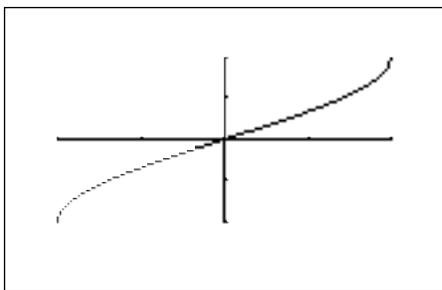
Method of Teaching

Use *Blackline Masters 5.1* and *5.2* to create overheads or handouts for demonstrating the computation and graphing of inverse trigonometric functions. Demonstrate the process in both radian and degree modes.

Next, use the *Blackline Master 5.3* to create a work sheet for the students. Have the students compute and graph inverse trigonometric functions. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 5.2

The following problems appear on **Blackline Master 5.2**. To compute inverse trigonometric functions you will turn the calculator on and press [⊞⊟⊠⊡] [CL] to enter computation mode. The computations can be done in degrees and radians. Let's do this one in radians by pressing [2ndF] [SET UP], touching **B DRG**, double touching **2 Rad**. Press [2ndF] [QUIT] to exit the menu. To find sin⁻¹ (.4), press [2ndF] [sin⁻¹] [•] [4] [ENTER]. The answer of .4115 radians is shown below. To find the result in terms of π press [÷] [2ndF] [π] [ENTER].

```
sin⁻¹ .4
              .411516846
Ans/π
              .13098988
```
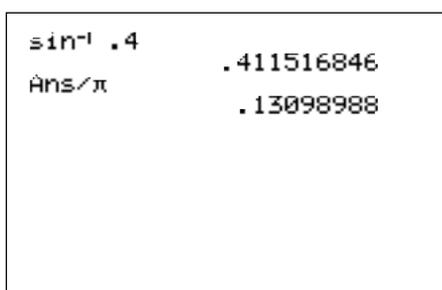
So, .4115 radians is equal to .13099π.

To graph the inverse cosine function, press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear the Y1 prompt. Enter $\cos^{-1} x$ for Y1 by pressing $\boxed{\text{2ndF}}$ $\boxed{\cos^{-1}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{ENTER}}$. Set your viewing window by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, and double touching **5 $\cos^{-1}$ x**.
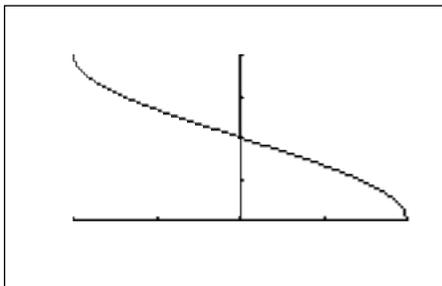
For Discussion

Students can use the calculators to discuss the mirror imagery of a trigonometric function and its inverse across $y = x$, and the composition of a trigonometric function with its inverse. Demonstrate to the students the location of inverse secant, inverse cosecant, and inverse cotangent within the MATH menu.

Additional Problems

Compute the following:

1. $\sin^{-1}(-.7) =$ _____ degrees

2. $\cos^{-1}(.5) =$ _____ degrees

3. $\tan^{-1}(-2) =$ _____ degrees

4. $\csc^{-1}(-2) =$ _____ degrees

5. $\sin^{-1}(.2) =$ _____ radians

6. $\cos^{-1}(-.6) =$ _____ radians

7. $\tan^{-1}(3) =$ _____ radians

8. $\cot^{-1}(3) =$ _____ radians

9. $\sec^{-1}(1.5) =$ _____ radians

Chapter six

SOLVING TRIGONOMETRIC EQUATIONS

Introducing the Topic

In this chapter, you and your students will learn how to solve trigonometric equations graphically. A trigonometric equation is an equation involving a trigonometric function. Usually, trigonometric equations are solved for the principal values for each function. These values are $-\pi/2 \le x \le \pi/2$ for sine and tangent, and $0 \le x \le \pi$ for cosine. The other solutions will then be the integral multiples of the function's period. Some trigonometric equations will have no solutions.

To solve an equation graphically, you will graph the left side of the equation as Y1 and the right side as Y2. In the case where the right side of the equation is zero, you will graph the left side as Y1 and solve for the intercepts. The point(s) of intersection will be the solutions to the equation.

Calculator Operations

To solve a trigonometric equation graphically, you will turn the calculator on and press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear Y1 of an old expression. Solving can be done in degrees and radians. Let's do the first one in degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu. To find the solutions of $3\sin^2 x - 4\sin x + 1 = 0$, press $\boxed{3}$ $\boxed{(}$ $\boxed{\sin}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{-}$ $\boxed{4}$ $\boxed{\sin}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{+}$ $\boxed{1}$ $\boxed{\text{ENTER}}$.

Set your viewing window to the principal values for sine by pressing WINDOW (–) 9 0 ENTER 9 0 ENTER 1 0 ENTER (–) 1 ENTER 1 ENTER 1 ENTER . Press GRAPH to view the graph.
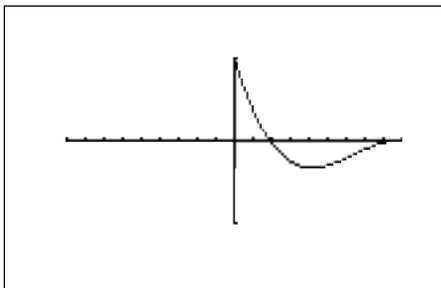
You will first learn how to zoom in to find a solution to the equation. You need to save this original graph and viewing window by pressing ZOOM , touching **G STO**, and double touching **1 StoWin**. This will allow you to return to this original screen after zooming in to find the first x intercept. Set the zoom factors by pressing ZOOM , touching **B FACTOR**, pressing ENTER 5 ENTER 5 ENTER . Press GRAPH to exit the menu and return to the graph. These larger factors will expedite the zooming in or zooming out that you desire to do.

To find the left intercept, press TRACE to engage the trace and move the cursor near this intercept by continuing to press the ▶ key. Your graph should look like the following one:
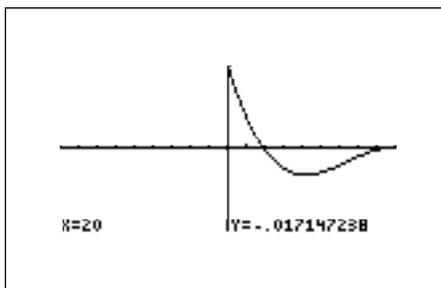
Now, press ZOOM , touch **A ZOOM**, and double touch **3 In** to zoom in on the left intercept and solution of this equation. Press TRACE and ◀ to trace near and approximate the intercept. As the graph below shows, the first solution is approximately 19.4°.
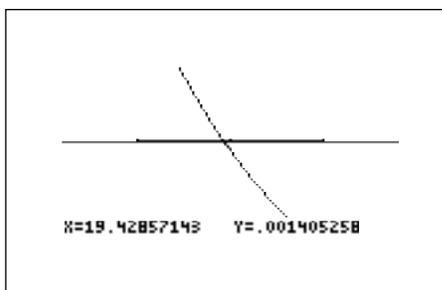
```
X=19.42857143    Y=.001405258
```

Return to the original screen by pressing $\boxed{\text{ZOOM}}$, touching **H RCL**, and double touching **1 RclWin**. To find the right intercept, press $\boxed{\text{TRACE}}$ and move the tracer near the intercept. Notice the intercept is 90°.


```
X=90              Y=0
```
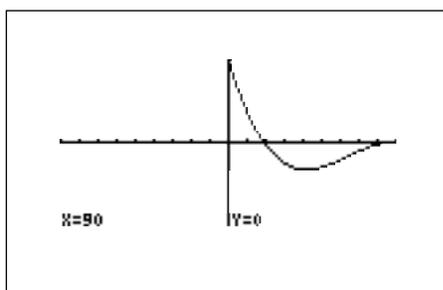
Method of Teaching

Use *Blackline Masters 6.1* and *6.2* to create overheads or handouts for demonstrating the solving of trigonometric equations graphically. Demonstrate both the solving by zooming described above in *Calculator Operations* and the CALC feature demonstrated in Using *Blackline Master 6.2*. In addition, solve equations in both radian and degree modes.

Next, use the *Blackline Master 6.3* to create a worksheet for the students. Have the students find the solutions to the trigonometric equations through zooming and calculation in both radian and degree modes. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 6.2

To solve a trigonometric equation graphically using the CALC feature of the Sharp graphing calculator, you will turn the calculator on and press $\boxed{Y=}$ $\boxed{CL}$ to access and clear Y1 of an old expression. Solving can be done in degrees and radians. Let's do this one in radians by pressing $\boxed{2ndF}$ $\boxed{SET\ UP}$, touching **B DRG**, and double touching **2 Rad**. Press $\boxed{2ndF}$ $\boxed{QUIT}$ to exit the menu.
To find the solutions of $2 \sin^2 x - \cos x - 1 = 0$, press $\boxed{2}$ $\boxed{(}$ $\boxed{\sin}$ $\boxed{X/\theta/T/n}$ $\boxed{)}$
$\boxed{a^b}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{-}$ $\boxed{\cos}$ $\boxed{X/\theta/T/n}$ $\boxed{-}$ $\boxed{1}$ $\boxed{ENTER}$.

Set your viewing window to the principal values for sine and cosine by pressing $\boxed{WINDOW}$ $\boxed{(-)}$ $\boxed{2ndF}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{ENTER}$ $\boxed{2ndF}$ $\boxed{\pi}$ $\boxed{ENTER}$ $\boxed{2ndF}$ $\boxed{\pi}$ $\boxed{\div}$
$\boxed{4}$ $\boxed{ENTER}$ $\boxed{(-)}$ $\boxed{1}$ $\boxed{ENTER}$ $\boxed{1}$ $\boxed{ENTER}$ $\boxed{1}$ $\boxed{ENTER}$. Press $\boxed{GRAPH}$ to view the graph.
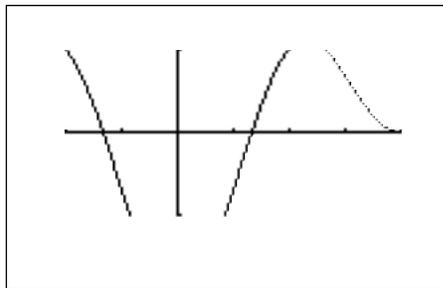
To "calculate" an intercept (solution), press $\boxed{2ndF}$ $\boxed{CALC}$ and double touch **5 X_Incpt**. You can calculate another intercept by pressing $\boxed{2ndF}$ $\boxed{CALC}$ and double touching **5 X_Incpt** again.
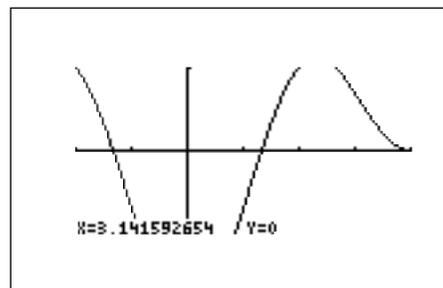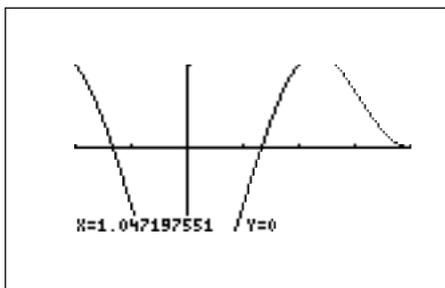
Pressing $\boxed{2ndF}$ $\boxed{CALC}$ and double touching **5 X_Incpt** will find the third solution.

X=-1.04719755   Y=0
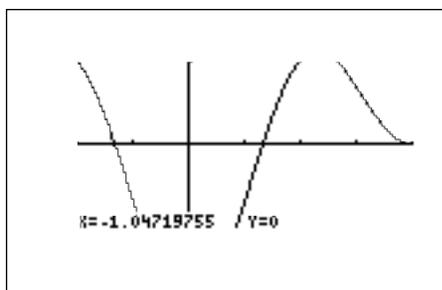
To solve a trigonometric equation with functions on both the left and right sides of the equality, you will press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear Y1 of an old expression. To find the solutions of $2\tan x \sin x + 2 \sin x = \tan x + 1$ you will enter the left side of the equality for Y1 and the right side for Y2. To enter the left side as Y1, press $\boxed{2}$ $\boxed{\text{tan}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{sin}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\text{sin}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{ENTER}}$. To enter the right side as Y2, press $\boxed{\text{tan}}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{+}$ $\boxed{1}$ $\boxed{\text{ENTER}}$. Set your viewing window to the combined principal values by pressing $\boxed{\text{WINDOW}}$ $\boxed{(-)}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{\text{ENTER}}$ $\boxed{(-)}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to view the graph.
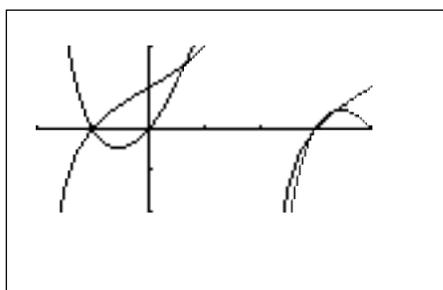
There appears to be four solutions to the equation. To "calculate" the intersections (solutions), press $\boxed{\text{TRACE}}$, move the tracer right until it appears on the screen, and press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **2 Intsct**. You can calculate the next intersection by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **2 Intsct** again. Shown below are two of the solutions (in radians) found by the CALC feature.
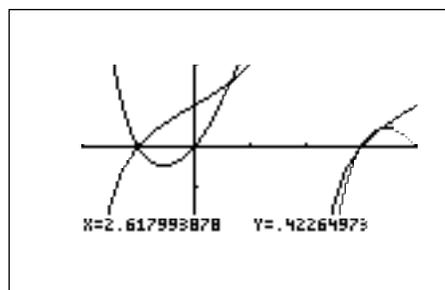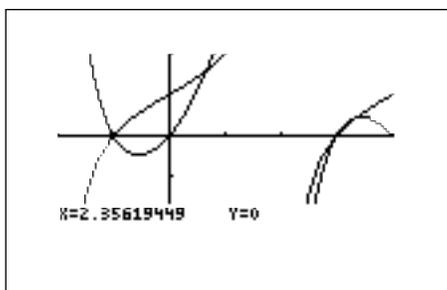
Calculate two more times to find the third and fourth solutions.
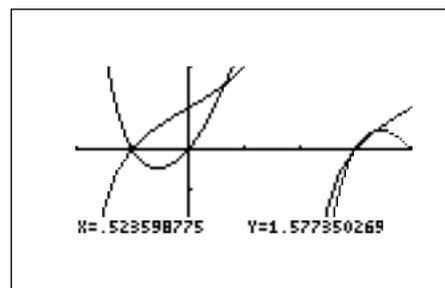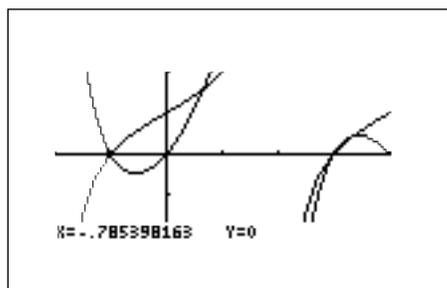
Note that 2.3562 radians is an integral multiple of ⁻.7854 radians. Therefore, there are three solutions ⁻.7854, .5236, and 2.618.

For Discussion
Students can use the calculators to verify the additional solutions at integral multiples of the function's period.

Additional Problems
Solve the following equations using either the zoom or CALC feature:

1. $2\cos^2 x - 5\cos x + 2 = 0$ (in degrees)

2. $\tan^2 x = \cot x$ (in degrees)

3. $2\cos^2 x = \sin x + 1$ (in degrees)

4. $3\tan^2 x + 4\sec x + 4 = 0$ (in radians)

5. $\cos x \tan x - \sin^2 x = 0$ (in radians)

6. $2\sin x \cos x + 4\sin x = \cos x + 2$ (in radians)

Chapter seven

APPLICATIONS WITH RIGHT TRIANGLES

Introducing the Topic

The earliest applications of trigonometry involved solving right triangles. In this chapter, you and your students will explore the use of graphics calculator, particularly the SOLVER feature, in finding the solution to problems involving applications with right triangles.

Calculator Operations

Before using the calculator to solve problems in this chapter, set the calculator to operate in degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up menu.

Historically, the six trigonometric functions were defined in relation to a right triangle.

$$\sin \theta = \frac{A}{C} \qquad\qquad \csc \theta = \frac{C}{A}$$

$$\cos \theta = \frac{B}{C} \qquad\qquad \sec \theta = \frac{C}{B}$$

$$\tan \theta = \frac{A}{B} \qquad\qquad \cot \theta = \frac{B}{A}$$

Given that A = 3 and C = 6, find B and θ using the SOLVER. To access the
SOLVER, press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, and press $\boxed{\text{CL}}$ to delete any equation that may
appear on the screen.

Type in the equation $\sin\theta = \dfrac{A}{C}$ with the keystrokes $\boxed{\sin}$ $\boxed{\text{ALPHA}}$ $\boxed{\theta}$ $\boxed{\text{ALPHA}}$
$\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{C}}$. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch
A METHOD, and double touch **1 Equation** to choose the Equation solver.
Press $\boxed{\text{ENTER}}$ and see the list of variables:

```
Solver:Equation
  θ=0
  A=0
  C=0
```

Press $\boxed{\blacktriangledown}$ to move the blinking cursor over the A and press $\boxed{3}$ $\boxed{\text{ENTER}}$. Type in the
value of C by pressing $\boxed{6}$ $\boxed{\text{ENTER}}$. Solve for θ by pressing $\boxed{\blacktriangle}$ $\boxed{\blacktriangle}$ $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$.

```
Equation solver

 θ=30
```

To use the SOLVER to find B, edit the current equation to the new equation cos
$\theta = \dfrac{B}{C}$. Do this by pressing $\boxed{\text{CL}}$ $\boxed{\text{CL}}$ to display the current equation $\sin\theta = \dfrac{A}{C}$.
With the cursor over the s in sin, press $\boxed{\cos}$ to insert cos. Press $\boxed{\text{DEL}}$ to delete
sin. Press $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleright}$ to move the cursor over the A in the numerator of the
fraction. Press $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ to insert B. Press $\boxed{\text{DEL}}$ to delete A.

Press ENTER to see the variable list. The value for θ and C will remain as before. Press ▼ to position the cursor over the B and press 2ndF EXE to find B.

```
Equation solver
   B=5.196152423
```

Method of Teaching

Use *Blackline Master 7.1* to create an overhead or handout for using the SOLVER in applications with right triangles. Next, use *Blackline Masters 7.2* and *7.3* to create worksheets for the students. Point out to the students that the SOLVER is an alternative to the methods of solution introduced in Chapter 5. Use the topics *For Discussion* to supplement the worksheets.

While in the SOLVER, press 2ndF SOLVER , touch **A METHOD** and notice that the SOLVER utilizes three methods of solving equations. The Equation method is discussed in this chapter. The Graphic and Newton methods will be discussed in Chapter 8.

Using Blackline Master 7.2

A rectangle has length 16 cm and width 8.6 cm. Find the degree measure of angle A. Determine the length of the diagonal of the rectangle.
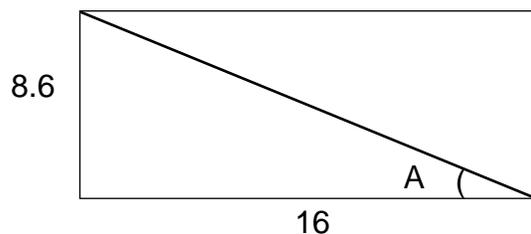
Press ⊞⊟⊠⊘ 2ndF SOLVER to access the SOLVER, and press CL to delete the equation that is on the screen. Enter the equation $\tan \theta = \dfrac{A}{B}$ by pressing tan

$\boxed{\text{ALPHA}}$ $\boxed{\theta}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$. Press $\boxed{\text{ENTER}}$ to view the variable list, and enter 8.6 for A and 16 for B. (The value of θ will be listed as the one that was used in the previously entered equation.) Position the cursor over θ and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find θ.

```
Equation solver
  θ=28.25802961
```

To determine the length of the diagonal of the rectangle, return to the equation by pressing $\boxed{\text{CL}}$ $\boxed{\text{CL}}$ and edit the current equation to read sin θ = $\dfrac{A}{C}$.

The value of θ should be as before, so enter 8.6 for A and solve for C.

```
Equation solver
  C=18.16480113
```

For Discussion

Students can edit a current equation in the SOLVER to a new equation instead of clearing the current equation and typing in the new equation.

Additional Problems

Problems 1-5 use the following figure:
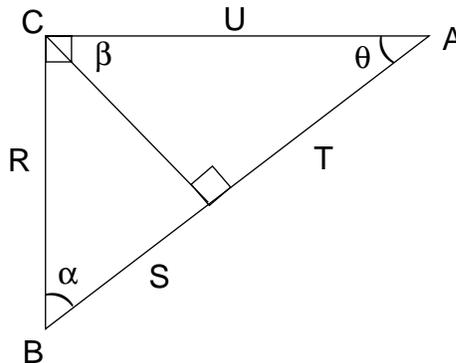
1. If θ=42° and U=8, find T.

2. If α=25° and U=16, find S+T.

3. If R=10 and S=8, find the degree measure of α.

4. If S=2, R=3.5, and U=4.8, find T.

5. Is it possible for the distance between points C and D to be 4 if S=2 and R=3.5? Explain why or why not.

6. A farmer wants to put a fence around the perimeter of a flat field that is in the shape of a right triangle. One angle of the triangle is 42° and the length of the side opposite this angle is 60 meters. Find the length of fencing that is required to enclose the field.

Chapter eight

LAW OF SINES AND LAW OF COSINES

Introducing the Topic

Chapter 7 considered techniques of finding the solution to problems involving right triangles. This chapter explores using the calculator with the Law of Sines and Law of Cosines. These two laws relate the angles and lengths of the sides of any triangle and have been used for many centuries in solving practical problems such as those that occur in map making, navigation, and surveying.

Calculator Operations

Before using the calculator to solve problems in this chapter, set the calculator to operate in degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up screen.

The Law of Sines states that the ratio of the lengths of the sides of a triangle to each other is the same as the ratio of the sines of the angles opposite the sides. That is, if A, B, and C are the measures of the three angles of a triangle and X, Y, and Z are respectively the lengths of the three sides opposite those angles, then:
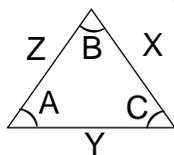
$$\frac{\sin A}{X} = \frac{\sin B}{Y} = \frac{\sin C}{Z}$$

You will find the SOLVER very useful when using the Law of Sines or Law of Cosines. To access the SOLVER, press 2ndF SOLVER , and press CL to delete any equation that may appear on the screen. To use the SOLVER with the Law of Sines, you can enter any of the following three equivalent statements:

$$\frac{\sin A}{X} = \frac{\sin B}{Y} \qquad\qquad \frac{\sin A}{X} = \frac{\sin C}{Z} \qquad\qquad \frac{\sin B}{Y} = \frac{\sin C}{Z}$$

Since you can label the sides and the angles of any triangle to correspond to one of these three statements, choose one, say $\frac{\sin A}{X} = \frac{\sin B}{Y}$, and enter it with the keystrokes:

a/b sin ALPHA A ▶ ALPHA X ▶ ALPHA = a/b sin ALPHA B ▶ ALPHA Y

Since you will be using this formula many times as you work through this chapter, store it into memory by pressing 2ndF SOLVER , touching **C SAVE**, pressing ENTER , and typing in a name of your choice, say LAWSINES by pressing L A W S I N E S . Note that the cursor is automatically set to ALPHA, so you can just type letters. Press ENTER to store the formula. The equation you have stored will reappear on the screen.

Press 2ndF SOLVER , touch **A METHOD**, double touch **1 Equation** to choose the Equation Solver. Press ENTER , and you see the list of variables for your equation.

```
Solver:Equation
   A=0
   X=0
   B=0
   Y=0
```
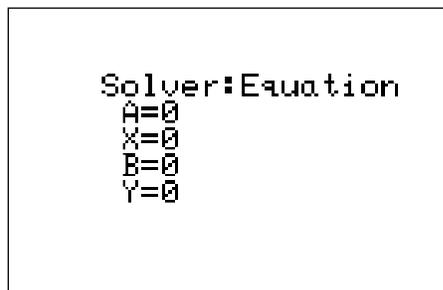
You may see values other than zero.
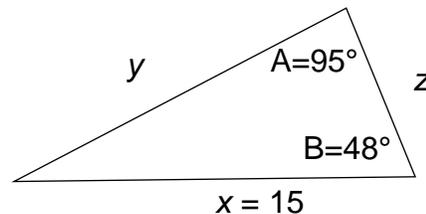
Consider the triangle shown on the right.
What is the value of *y*? What is the value of *z*?
With the blinking cursor over the A, type in 95 and press ENTER .

y   A=95°   *z*

B=48°

x = 15

Next type 15 for *x*, the length of the side opposite the 95°, and press ENTER .
Enter 48 for B in the same manner. Solve for *y* by positioning the blinking cursor
over the *y* and press 2ndF EXE . The *y* turns out to be about 11.19.

Use the fact that the sum of the angles in any triangle is 180° to find 37° as the
measure of the angle opposite side *z*. You can now apply methods in Chapter 7
to edit the current equation to one involving a known side and side *z*, say
$\frac{\sin A}{X} = \frac{\sin C}{Z}$, or you can work with the current equation using B = 37 and
y = *z*. In either case, you should find *z* = 9.06.

This problem could also be solved using the Graphic Solver. Press 2ndF
SOLVER , touch **A METHOD**, double touch **3 Graphic** to choose the Graphic
Solver. Enter the known quantities for the first part of this example: angle A
measures 95°, side *x* is of length 15, and angle B measures 48°. (To start the
problem from the beginning, enter 0 for *y*). Use ▼ to position the cursor over
the unknown quantity, *y*. To begin the solution process press 2ndF EXE and
you will see:

```
Graphic solver
variable range

BEGIN=0

END=1
```

You must enter the range values for the unknown (from BEGIN to END) within
which you expect to find the solution. Let the BEGIN = 5 by pressing 5 ENTER .
Enter a value for END that you do not expect *y* to exceed, say 25, by pressing
2 5 ENTER . Press 2ndF EXE and an autoscaled plot will be drawn
showing graphs of the left and right sides of the current equation.

The calculator will now display a blinking box in the upper right-hand corner of
the viewing screen while it looks for the intersection of the two graphs. If one is
found, the solution is displayed at the bottom of the screen.

Y=11.18975277
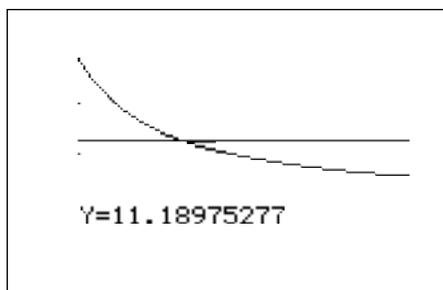
Consider another example. Suppose the length of the side of a triangle opposite a 65° angle is 4 and another side is of length 12.

Find the measure of the angle opposite the side of length 12. Press $\boxed{\text{CL}}$ to view the variable list. Enter A = 65, x = 4, and y = 12. Position the cursor on B and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to solve for the unknown angle. Since any angle of a triangle is between 0° and 180°, enter these values for BEGIN and END.

Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ and you will see that the graphs of the left- and right-hand sides of the equation $\sin\dfrac{A}{x} = \sin\dfrac{B}{y}$ do not intersect.

The message "No solution" in window is then displayed at the bottom of the screen. If you get this message when using the Graphic Solver, you should enter another BEGIN and/or END within which the Graphic Solver will search for solutions. In this case, however, you have included from BEGIN to END all possible values for the measure of angle B. Thus, there is no triangle that could have the indicated measures.

You can algebraically confirm this fact by noticing that sin B = (Y sin A)/X= 3 sin 65° = 2.719 >1 which is of course, impossible.

The Law of Cosines is helpful in solving problems when the lengths of two sides and the angle between those sides is known or when the lengths of all three sides are known. This law says that the square of the length of any one side of a triangle equals the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides and the cosine of the angle between those sides. That is, if x, y, and z are the lengths of the three sides of a triangle and θ is the angle between the two sides of lengths x and y, then:

$$z^2 = x^2 + y^2 - 2xy \cos\theta$$

Access the SOLVER with $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$ (or press $\boxed{\text{CL}}$ twice if you are still in the SOLVER). Press $\boxed{\text{CL}}$ to delete the current equation. Return to the Equation Solver by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation**. Next enter the Law of Cosines with the keystrokes:

$\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{x^2}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{x^2}$ $\boxed{-}$
$\boxed{2}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{\times}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{\times}$ $\boxed{\cos}$ $\boxed{\text{ALPHA}}$ $\boxed{\theta}$

Store this equation in the calculator using the method outlined to store the Law of Sines.

Suppose you are given a triangle in which two sides of lengths 13 cm and 8 cm enclose a 50° angle. Find the length of the third side of the triangle.

To find the solution using the Law of Cosines, press $\boxed{\text{ENTER}}$ to view the list of variables: Z, X, Y, and θ. Press $\boxed{\blacktriangledown}$ to position the cursor on X and enter 13. Next, enter 8 for Y and 50 for θ. Press $\boxed{\blacktriangle}$ to move the cursor over Z and press $\boxed{0}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to solve for Z. Notice that the Equation Solver has automatically changed to the Newton Solver.

```
Newton solver
START=0
STEP=.001
```

The calculator has chosen this method of solution since the equation is such that a simple rearrangement of the variables will not produce a value for the unknown. The Newton method uses iterations to approximate the root of the equation. You should supply an initial estimate of the solution in the START position (0 is the default value) and a step size STEP (.001 is the default value) that the SOLVER uses to determine successive estimates.

Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the solution.

```
Newton solver

 Z=9.964947425

RIGHT=99.30017719
LEFT=99.30017719
L-R=0
```

(If you receive an error message while using the Newton Solver, set the START value at your estimate of the solution and try slightly increasing the STEP value.) The method is continued until the left side and right side of the equation are within a defined tolerance.

Since the left and right sides of the equation are displayed as equal values, and the L – R = 0, the NEWTON SOLVER has found a solution that is correct to the limits of the calculator's accuracy.

Method of Teaching

Use *Blackline Master 8.1* to create an overhead or handout for using the Graphic Solver with the Law of Sines. Use *Blackline Master 8.2* to create an overhead or handout for using the Newton Solver with the Law of Cosines. Demonstrate the procedure to store each of these equations in the calculator memory for later recall.

Next, use *Blackline Master 8.3* to create a worksheet for the students. Remind them that some problems may be solved by a variety of methods. If information is not available to obtain a good estimate of START in the Newton Solver, students may find it more convenient to work with the Graphic Solver.

Use the topics *For Discussion* to supplement the worksheets. Retrieving (loading) and renaming equations that have been stored in the calculator is discussed in the owner's manual.

Using Blackline Master 8.3

The problems discussed in the *Calculator Operations* section are presented on *Blackline Masters 8.1* and *8.2*. Problem 1 on *Blackline Master 8.3* involves the angles of elevation of the sun. Remind students that the angle of elevation is the angle, measured upward from the horizontal, to the line of sight of an object. The Graphic Solver provides an added exploration when finding the solution to this problem. In the variable list for the Law of Sines, enter 60 for A, 50 for X, and 40 for Y. Solve for B using BEGIN= 0 and END= 180 to obtain B = 43.85°. Notice, however, that two intersections are visible. Press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and the Graphic Solver will find the other intersection at B =136.15°. Have students discuss why this is not a possible value for B. Once B is found, $90 - \theta = 180 - (60 + 43.85) = 76.15$ yielding $\theta = 13.85°$.

For Discussion

Students can discuss how to retrieve and rename the stored equations for the Law of Sines and the Law of Cosines.

Additional Problems

Problems 1-3 refer to the following situation:

A tower on the top of a hill is 40 feet tall. The angle of elevation from point B at the base of the hill to the top of the tower (point T) is 45°. The angle of elevation from point A, 800 feet from the base of the hill, to point T is 22°.
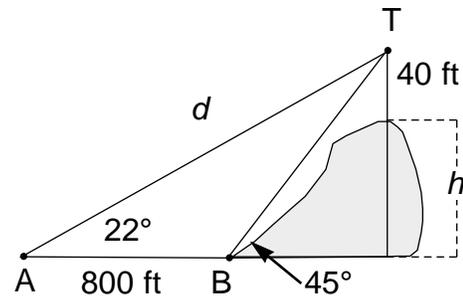
1. Find the distance *d* from point A to the top of the tower.

2. A cable is stretched taunt between points B and T. Find the length of the cable.

3. Find the height *h* of the hill.

4. The angle of elevation of the top of a flagpole from point A is 22°. The angle of depression of the top of the flagpole from a helicopter hovering 1200 feet above point A is 25°. Find the distance from the helicopter to the top of the flagpole.
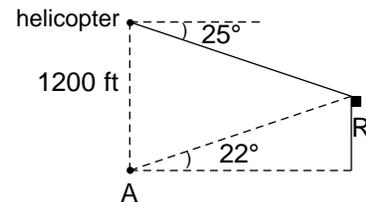
Chapter nine

AREA OF A TRIANGLE

Introducing the Topic

In this chapter, you and your students will explore three formulas and the use of the graphing calculator, particularly the SOLVER, in finding the area of a triangle.

Calculator Operations

Before using the calculator to solve problems in this chapter, set the calculator to operate in degrees with $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up screen.

Depending on the information given in the problem situation, there are three formulas you can use to find the area of any triangle:

1. The area of a triangle is one-half the product of the base and the height where the height is the length of the line segment drawn from a vertex of the triangle perpendicular to the opposite side called the base.
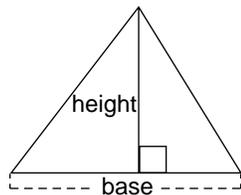
$$area = \frac{1}{2} \ base \bullet height$$

2. The area of a triangle is one-half the product of the lengths of two sides of the triangle and the measure of the angle included between those sides.
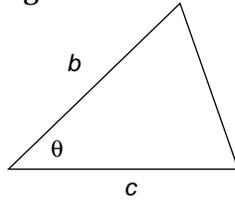
$$area = \frac{1}{2} bc \sin \theta$$

3. In the second half of the first century A.D., Heron of Alexandria stated the following formula for the area of any triangle in terms of the lengths of its sides. It is known as Heron's formula (or Hero's formula):
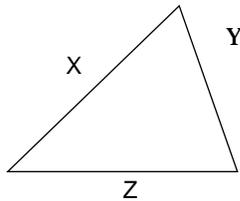
$$area = \sqrt{[S(S\text{-}X)(S\text{-}Y)(S\text{-}Z)]}$$
where $S = \frac{1}{2}(X + Y + Z)$.

The first formula for the area of a triangle, area = $\frac{1}{2}$ base • height, can be typed as A = $\frac{1}{2}$ B•H. Note that the multiplication key $\boxed{\times}$ must be pressed between the base and height to separate the variable names. To access the SOLVER, press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, and press $\boxed{\text{CL}}$ to delete any equation on the screen.

Type in the equation A = $\frac{1}{2}$ B • H with the keystrokes: $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{1}$ $\boxed{\blacktriangleright}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\times}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{H}}$.

$$A= \frac{1}{2} \ B*H$$

Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation** to choose the Equation solver. Press $\boxed{\text{ENTER}}$ to see the list of variables for your equation:

```
Solver:Equation
  A=0
  B=0
  H=0
```

To find the area of a triangular garden plot with base = 4 meters and height = 8.35 meters, use ▼ to move the blinking cursor over B and type in 4. Type in 8.35 for H and press ENTER . Press ▲ twice to move the prompt to the A variable. Press 2ndF EXE to find the area, 16.7 square meters. (You may wish to save this formula for later recall using the methods described in Chapter 7.)

```
Equation solver
  A=16.7
```

The second formula for the area of a triangle can be typed in as A = $\frac{1}{2}$B•C•sin θ. Press CL three times to return to and delete the current equation.

Type in the equation A = $\frac{1}{2}$B•C•sin θ with the keystrokes: ALPHA A ALPHA = a/b 1 ▶ 2 ▶ ALPHA B × ALPHA C × sin ALPHA θ . Consider the triangle below where sides of lengths 4.8 and 5.73 enclose a 47° angle. The height of this triangle is not known, but you do have the necessary information to use the formula A = $\frac{1}{2}$B•C•sin θ.
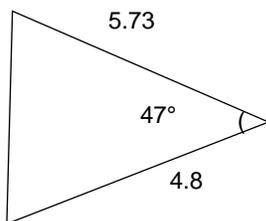
5.73
47°
4.8

Press ENTER to access the variable list, enter 5.73 for B, 4.8 for C (or vice-versa), and 47 for θ. Move the cursor to the A variable. Press 2ndF EXE to solve for the area.

```
┌─────────────────────────────┐
│ Equation solver             │
│   A=10.0575761              │
│                             │
│                             │
│                             │
│                             │
│                             │
└─────────────────────────────┘
```

Method of Teaching

Use *Blackline Master 9.1* to create an overhead or handout for using the SOLVER in applications with areas of triangles. Remind the students to store each of the formulas under a name of their choice before deleting the current formula and entering a new one. Next, use *Blackline Masters 9.2, 9.3,* and *9.4* to create work-sheets for the students. Encourage different methods of solution and use of both the Equation solver and Graphic solver. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 9.2

The second problem presented under the CALCULATOR OPERATIONS section, also provides a nice exercise with the Graphic solver. Press 2ndF SOLVER , touch **A METHOD**, and double touch **3 Graphic** to choose the Graphic solver.

To start the problem from the beginning, set A to 0. Move the cursor to the A variable and press 2ndF EXE to see the Graphic solver variable range screen. Recall that the Graphic solver searches for the intersection of the left and right hand sides of your equation from BEGIN to END.

Since area is always nonnegative, 0 could be used for BEGIN. Enter END a value that you do not expect area to exceed, say 25. (A little bit of estimation helps here! If your value for END is too small, increase it and try again.)

Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ and an autoscaled graph of the two sides of the equation is drawn and the solution is obtained.

A=10.0575761
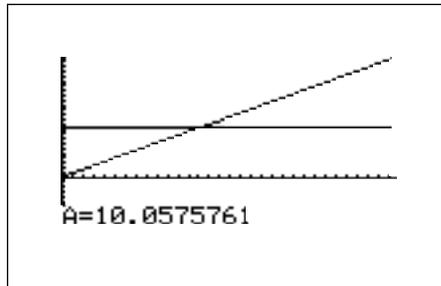
Another formula for area of a triangle is Heron's formula. Enter Heron's formula, A = √ [S(S-X)(S-Y)(S-Z)], press $\boxed{\text{CL}}$ three times, and type the following: $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{=}}$ $\boxed{\text{2ndF}}$ $\boxed{\sqrt{}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{\text{(}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{\text{–}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{\text{)}}$ $\boxed{\text{(}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{\text{–}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{\text{)}}$ $\boxed{\text{(}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{\text{–}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{\text{)}}$.

A=√S(S-X)(S-Y)(S-Z)

(Note that the square root symbol extends over the entire right hand side of the equation, automatically grouping the expression under it.)

You could enter the semiperimeter, S = $\frac{1}{2}$ (X + Y + Z), in a separate formula in the SOLVER, but it is probably easier to find the value of S in the computation mode and then use Heron's formula in the SOLVER. Press $\boxed{\text{ENTER}}$ to view the variable list.

For Discussion
Students can discuss the steps to enter the distance formula in the SOLVER.

Additional Problems

Problems 1-3 refer to the figure below:
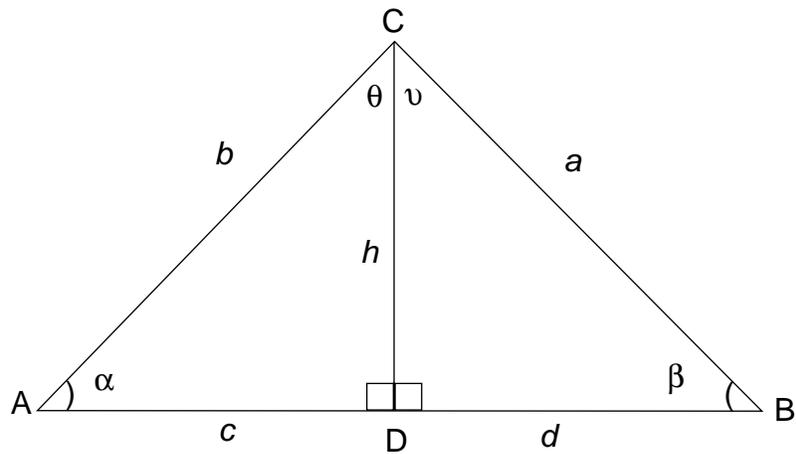
1. If $\alpha = 60°$, $c = 7$ and $b = 10$, find the area of triangle ACD.

2. If $a = 2.5$, $d = 1.5$ and $h = 2$, find the area of triangle DCB.

3. If $h = 14.68$, $c = 16$, and $d = 10$, find the area of triangle ACB.

4. Find the area of a triangle with vertices at (ˉ1, 5), (1, ˉ1) and (4, 2).

5. Suppose you are told that a triangle with two sides of length 12 and 15 has an area equal to 200 and you are asked to find the measure of the angle between the two given sides. What would you find? Why did you find it?

Chapter ten

AREA OF A CIRCULAR SECTOR

Introducing the Topic

Let's explore one of the many uses for angles measured in radians, and that is determining the area of a sector of a circle.

Calculator Operations

Before using the calculator to solve problems in this chapter, set the calculator to operate in radians by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up screen.

A portion of a curve is called an arc. When the curve is a circle, a *central angle* is an angle whose vertex is at the center of the circle. The central angle is said to intercept, or *subtend*, the arc.
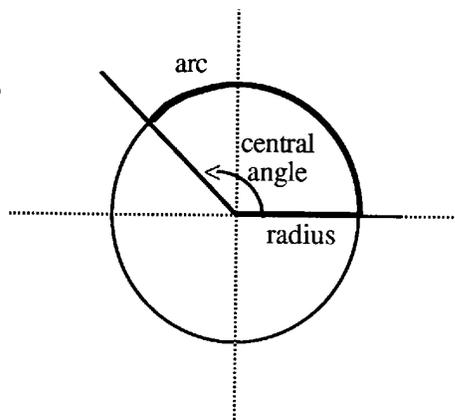
The portion of the circle and its interior that is formed by the two radii enclosing a central angle and an arc is called a *sector* of the circle. The formula for the area of a circular sector in terms of the radius r of the circle and the central angle θ, measured in radians, is area = $\frac{1}{2} r^2 \theta$.
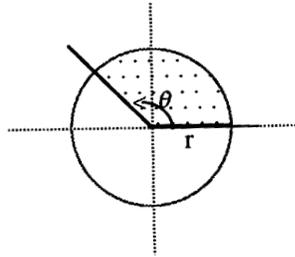
To enter this formula in the calculator, access the SOLVER by pressing [⊞⊟⊠⊞].
[2ndF] [SOLVER] . Press [CL] to delete any equation that appears on the screen.
Type in the equation A= $\frac{1}{2}$ R²θ by pressing [ALPHA] [A] [ALPHA] [=] [1]
[a/b] [2] [▶] [ALPHA] [R] [x²] [ALPHA] [θ] .

$$A = \frac{1}{2} R^2 \theta$$

Consider this problem situation. A lawn sprinkler is set so that the region of the lawn that is watered is approximately a sector of a circle of radius 6 feet. If the central angle of the sector is $\frac{\pi}{5}$ radians, find the area of the region that is watered by the sprinkler.

6 ft

6 ft

$\frac{\pi}{5}$
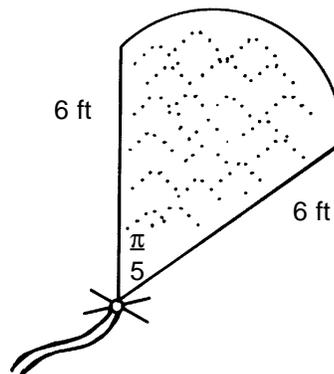
To find the area using the Equation solver, first press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation**. Press $\boxed{\text{ENTER}}$ and see the list of variables: A, R and θ. Use $\boxed{\blacktriangledown}$ to place the cursor over R, type in 6, and press $\boxed{\text{ENTER}}$. Type in θ by pressing $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{\text{ENTER}}$. Press $\boxed{\blacktriangle}$ twice to place the cursor on the A variable. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area in square feet.

```
Equation solver
  A=11.30973355
```

To find the area using the Newton solver, first press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **2 Newton**. Reset A to 0 to start the problem from the beginning. The values of R and θ are correct for this problem, so use $\boxed{\blacktriangle}$ to place the cursor over the A variable and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to view the Newton solver screen.

Since area is never negative, you could set START=0. Leave the STEP size as .001 and remember that you should change it to a slightly larger value if an error message results. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area in square feet.

```
Newton solver
  A=11.30973355

RIGHT=11.30973355
LEFT =11.30973355
L-R  =0
```

Method of Teaching

Use *Blackline Masters 10.1* and *10.2* to create overheads or handouts for using the SOLVER in applications with the area of a circular sector. Demonstrate the steps to prepare the calculator for use with the SOLVER, and emphasize the fact that the central angle in the formula for the area of a sector of a circle must be in radians, not degrees. Encourage students to give units with all answers. Next, use *Blackline Master 10.3* to create a worksheet for finding the area of a circular sector. Encourage students to explore all three techniques (Equation, Newton, and Graphic) available in the SOLVER so they will have experience with other methods if the one they are attempting to use fails. Use the topics *For Discussion* provided to supplement the worksheets.

Using Blackline Master 10.2

To find the area using the Graphic Solver, press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **3 Graphic**. Reset A to 0 to start the problem from the beginning. Place the cursor on the A variable and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$. Recall that BEGIN and END are the lower and upper values between which you feel the solution will lie. Enter a 20 for END so that the intersection of the two graphs is visible on the screen.
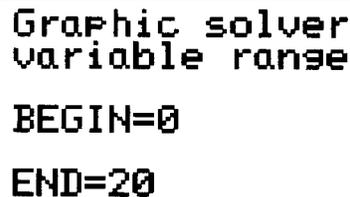
```
Graphic solver
variable range

BEGIN=0

END=20
```

Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area.

```
A=11.30973356
```
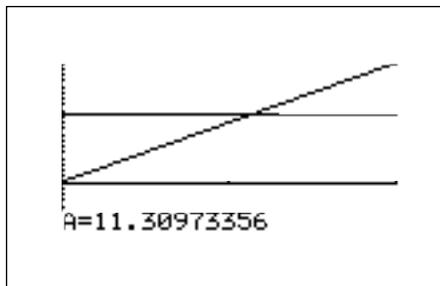
For Discussion

Students can discuss the advantages and/or disadvantages of using the three calculator solvers: Equation solver, Newton solver, and Graphic solver in various problem situations.

Additional Problems

1. A circle has radius 6 cm. Find the area of a sector of this circle whose arc subtends an angle of measure $\frac{3\pi}{4}$ radians.

2. A circular garden has diameter 8 feet. Broccoli is to be planted in a sector of this garden whose arc subtends an angle of 50°. Find the area of the broccoli sector.

3. If the area of a sector of a circle is 7.52 square inches and the radius of the circle is 2.4 inches, find the measure of the angle subtended by the arc of the sector. (Give units with your answer.)

CONTENTS OF
REPRODUCIBLE BLACKLINE MASTERS

Use these reproducible Blackline Masters to create handouts, overhead transparencies and activity worksheets.

1.1

GRAPHS OF TRIGONOMETRIC FUNCTIONS

Steps for graphing the sine trigonometric function

1. Turn the calculator on and press $\boxed{Y=}$.

2. Press $\boxed{CL}$ to remove an old Y1 expression.

3. Press $\boxed{\blacktriangledown}$ $\boxed{CL}$ to clear additional expressions.

4. To set up the calculator press $\boxed{2ndF}$ $\boxed{SET\ UP}$, touch **B DRG**, double touch **2 Rad**, touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**.

5. Press $\boxed{2ndF}$ $\boxed{QUIT}$ again to return to the Y prompts.

6. Enter the sine function ($y = \sin x$) for Y1, by pressing $\boxed{\sin}$ $\boxed{X/\theta/T/n}$.

7. Enter the viewing window range by pressing $\boxed{ZOOM}$, touching **E TRIG**, and double touching **1 sin x**.

Steps for graphing the cosecant trigonometric function.

1. Press $\boxed{Y=}$ $\boxed{CL}$ to remove an old Y1 expression.

2. Enter the cosecant function for Y1 by pressing $\boxed{MATH}$, touch **A CALC**, double touch **10 csc**, and press $\boxed{X/\theta/T/n}$.

3. Continue to use the sine function's built-in viewing range.

4. Press $\boxed{GRAPH}$ to view the graph.

1.2

GRAPHS OF TRIGONOMETRIC FUNCTIONS

Activity 1   Graph the cosine function.

1. Turn the calculator on and press $\boxed{\text{Y=}}$.

2. Press $\boxed{\text{CL}}$ to remove an old Y1 expression.

3. Press $\boxed{\blacktriangledown}$ $\boxed{\text{CL}}$ to clear additional expressions.

4. To set up the calculator press $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, double touch **2 Rad**, touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**.

5. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ again to return to the Y prompts.

6. Enter the cosine function ($y = \cos x$) for Y1, by pressing $\boxed{\text{cos}}$ $\boxed{\text{X/}\theta\text{/T/}n}$.

7. Enter the viewing window range by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, and double touching **2 cos x**.

8. Sketch your graph in the box.
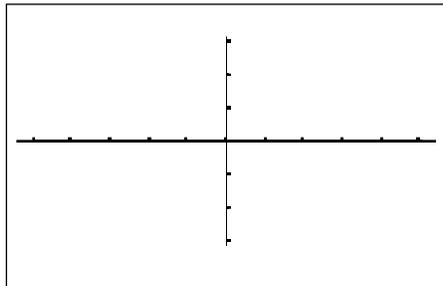
1.3

GRAPHS OF TRIGONOMETRIC FUNCTIONS

Activity 2   Graph the tangent function.

1. Turn the calculator on and press $\boxed{\text{Y=}}$.

2. Press $\boxed{\text{CL}}$ to remove an old Y1 expression.

3. Press $\boxed{\blacktriangledown}$ $\boxed{\text{CL}}$ to clear additional expressions.

4. To set up the calculator press $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, double touch **2 Rad**, touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**.

5. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ again to return to the Y prompts.

6. Enter the cosine function ($y = \tan x$) for Y1, by pressing $\boxed{\text{tan}}$ $\boxed{\text{X/}\theta\text{/T/}n}$.

7. Enter the viewing window range by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, and double touching **3 tan x**.

8. Sketch your graph in the box.
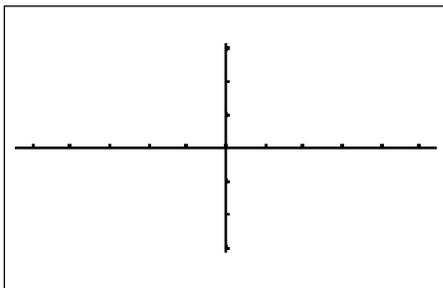
1.4

GRAPHS OF TRIGONOMETRIC FUNCTIONS

Activity 3   Graph the secant function.

1. Press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to remove an old Y1 expression.

2. Enter the secant function for Y1 by pressing $\boxed{\text{MATH}}$, touch **A CALC**
    double touch **09 sec**, and press $\boxed{\text{X/}\theta\text{/T/}n}$.

3. Use the cosine function's built-in viewing range by pressing $\boxed{\text{ZOOM}}$,
    touching **E TRIG**, and double touching **2 cos x**.
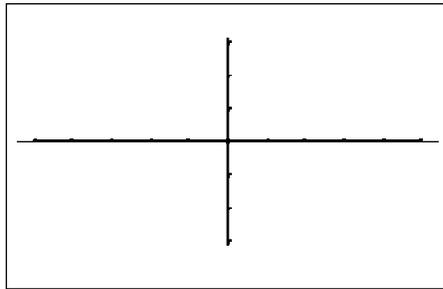
Activity 4   Graph the cotangent function.

Try graphing the cotangent function without looking at the steps provided in
Activities 1-3. Use the built-in viewing window range for tangent and sketch your
graph in the box below.
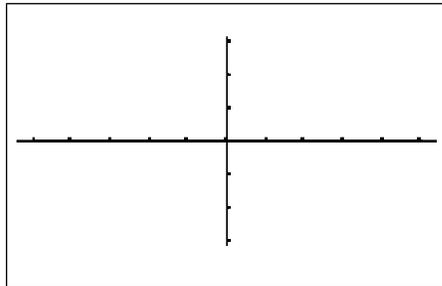
2.1

EVALUATING TRIGONOMETRIC FUNCTIONS

Steps for evaluating sin 45° directly

1. Turn the calculator on and press ⊞⊟⊠⊞ to enter computation mode.

2. Press $\boxed{\text{CL}}$ to clear the viewing window of any old expressions.

3. Set the calculator to operate in degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$ touch **B DRG**, and double touch **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu.

4. Enter and evaluate the expression by pressing $\boxed{\text{sin}}$ $\boxed{4}$ $\boxed{5}$ $\boxed{\text{ENTER}}$.

Steps for approximating sin ($\frac{\pi}{3}$) graphically

1. Press $\boxed{\text{Y=}}$ followed by $\boxed{\text{CL}}$ to clear an old expression for Y1.

2. Enter the function sin x by pressing $\boxed{\text{sin}}$ $\boxed{\text{X/}\theta\text{/T/}n}$. Set the calculator to radian mode by pressing $\boxed{\text{2nd F}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **2 Rad**. Press $\boxed{\text{2nd F}}$ $\boxed{\text{QUIT}}$ to exit the menu.

3. View the graph by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, double touching **1 sin x**.

4. Now, engage the trace by pressing $\boxed{\text{TRACE}}$ and tracing near $x = \frac{\pi}{3} = 1.047$.

5. With the cursor located near $x = 1.047$, you can zoom in to find a better approximation for the y value. Press $\boxed{\text{ZOOM}}$, touch **A ZOOM**, and double touch **3 In** to zoom in on the cursor.

6. Once again, trace over near $x = 1.047$. Zoom in one more time to see an even better approximation.

2.2

EVALUATING TRIGONOMETRIC FUNCTIONS

Steps for evaluating sec ($\frac{\pi}{4}$) directly in degree mode

1. Press ⊞⊟ to enter computation mode.

2. Press 2ndF SET UP to verify the calculator is in degree mode. If it is not, change it to degree mode. Press 2ndF QUIT to exit the menu.

3. Enter and evaluate the expression by pressing MATH , touching **A CALC**, double touching **09 sec**, pressing 2ndF π a/b 4 ▶ MATH , touching **E ANGLE**, double touching **4 r**, and pressing ENTER .

Steps for approximating tan 20° graphically

1. Press Y= and press CL to remove an old expression for Y1.

2. Press 2ndF SET UP to verify the calculator is in degree mode. If it is not, change it. Press 2ndF QUIT to exit the menu.

3. Enter the function tan x for Y1 by pressing tan X/θ/T/n .

4. Instead of graphing the function in the default viewing window for tangent, press WINDOW adjust the Xmin and Xmax to 10° and 30° (value of interest ± 10°) respectively to start out closer to the value of interest. Set the Xscl = 1. Press ZOOM , touch **A ZOOM**, and double touch **1 Auto** to view the graph.

5. Now, engage the trace by pressing TRACE . Notice the tracer appears at x = 20° since it is the middle of the x values.

2.3

EVALUATING TRIGONOMETRIC FUNCTIONS

Activity 1   **Direct evaluation of trigonometric functions in degrees**

1. sin 80°

2. cos (⁻10°)

3. tan 70°

4. csc 55°

5. sec 22°

6. cot 40°

Activity 2   **Direct evaluation of trigonometric functions in radians**

7. sin $(-\frac{\pi}{5})$

8. cos $(\frac{7\pi}{6})$

9. tan $(\frac{\pi}{4})$

10. csc $(\frac{5\pi}{4})$

11. sec $(\frac{2\pi}{3})$

12. cot $(-\frac{\pi}{7})$

2.4

EVALUATING TRIGONOMETRIC FUNCTIONS

**Activity 3   Graphical approximation of trigonometric functions
in degrees**

13. sin 18°

14.   cos 33°

15.   tan (‾134°)

16. csc 85°

17.   sec 145°

18.   cot 235°

**Activity 4   Graphical approximation of trigonometric functions
in radians**

19. sin ($\frac{4\pi}{5}$)

20.   cos ($\frac{6\pi}{7}$)

21.   tan ($\frac{3\pi}{4}$)

22. csc ($\frac{\pi}{9}$)

23.   sec ($\frac{3\pi}{8}$)

24.   cot ($\frac{5\pi}{11}$)

3.1

GRAPHICAL VERIFICATION OF FORMULAS AND IDENTITIES

1. Verify the double angle formula $\sin 2\theta = 2 \sin \theta \cos \theta$, by graphing Y1 = sin (2x) and Y2 = 2 sin x cos x and show that they are in fact the same graph.

2. Turn the calculator on and press $\boxed{\text{Y=}}$. Press $\boxed{\text{CL}}$ to clear Y1 of an old expression. Press $\boxed{\blacktriangledown}$ $\boxed{\text{CL}}$ to clear additional expressions.

3. Make sure the calculator is in radian mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu.

4. Now enter sin (2x) for Y1 by pressing $\boxed{\text{sin}}$ $\boxed{(}$ $\boxed{2}$ $\boxed{X/\theta/T/n}$ $\boxed{)}$ $\boxed{\text{ENTER}}$ and the 2 sin x cos x for Y2 by pressing $\boxed{2}$ $\boxed{\text{sin}}$ $\boxed{X/\theta/T/n}$ $\boxed{\text{cos}}$ $\boxed{X/\theta/T/n}$ $\boxed{\text{ENTER}}$.

5. Set your viewing window by pressing $\boxed{\text{WINDOW}}$ $\boxed{\text{EZ}}$, touch **5**, touch ☙ on the screen, touch **6 ⁻2π<X,<2π**, touch ☙on the screen, and touch **5 ⁻5<Y<5**.

3.2

GRAPHICAL VERIFICATION OF FORMULAS AND IDENTITIES

Verify the half angle formula $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{(1 + \cos\theta)}{2}}$:

1. To verify the formula, you will graph $Y1 = \cos\frac{x}{2}$ and $Y2 = \pm\sqrt{\frac{(1 + \cos x)}{2}}$ and show that they are in fact th same graph.

2. In this problem you will need to limit your domain or x values to intervals where cosine is either negative or positive and enter the appropriate radical for Y2.

3. To do this you will turn the calculator on and press $\boxed{\text{Y=}}$. Press $\boxed{\text{CL}}$ to clear Y1 of an old expression.

4. Now, enter the $\cos\frac{x}{2}$ for Y1 by pressing $\boxed{\cos}$ $\boxed{\text{X/θ/T/}n}$ $\boxed{\text{a/b}}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ and the $\sqrt{\frac{(1 + \cos x)}{2}}$ for Y2 (limiting to where cosine is positive) by pressing $\boxed{\text{CL}}$ $\boxed{\text{2ndF}}$ $\boxed{\sqrt{}}$ $\boxed{\text{a/b}}$ $\boxed{1}$ $\boxed{+}$ $\boxed{\cos}$ $\boxed{\text{X/θ/T/}n}$ $\boxed{\blacktriangledown}$ $\boxed{2}$ $\boxed{\text{ENTER}}$.

5. Cos $\left(\frac{x}{2}\right)$ will be positive for a viewing window of $0 < x < \pi$. Enter this viewing window by pressing $\boxed{\text{WINDOW}}$ $\boxed{0}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{(-)}$ $\boxed{3}$ $\boxed{\text{ENTER}}$ $\boxed{3}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$.

6. Press GRAPH to view the graphs. Notice they appear as one graph.

7. To verify the formula for where cosine is negative, press $\boxed{\text{Y=}}$ $\boxed{\blacktriangledown}$ to access the Y2 prompt and press $\boxed{(-)}$ to insert a negative in front of the radical. Cos $\left(\frac{x}{2}\right)$ will be negative for a viewing window of $\pi < x < 3\pi$. Enter this viewing window by pressing $\boxed{\text{WINDOW}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$ $\boxed{3}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to view the graphs.

3.3

GRAPHICAL VERIFICATION OF FORMULAS AND IDENTITIES

Activity 1   **Verify the half-angle formula**
$$\sin \left(\frac{\theta}{2} \right) = \pm \sqrt{\left(\frac{1 - \cos \theta}{2} \right)}:$$

1. Press $\boxed{Y=}$ $\boxed{CL}$ to clear Y1 of an old expression. Press $\boxed{ENTER}$ $\boxed{CL}$ to clear additional expressions.

2. Make sure the calculator is in radian mode by pressing $\boxed{2ndF}$ $\boxed{SET\ UP}$, touching **B DRG**, and double touching **2 Rad**. Press $\boxed{2ndF}$ $\boxed{QUIT}$ to exit the set-up menu.

3. Enter the sin ($\frac{X}{2}$) for Y1 by pressing $\boxed{\sin}$ $\boxed{X/\theta/T/n}$ $\boxed{a/b}$ $\boxed{2}$ $\boxed{ENTER}$.

4. Enter the $\sqrt{\left(\frac{1 - \cos x}{2} \right)}$ for Y2 (limiting domain to an interval where sine is positive) by pressing $\boxed{2ndF}$ $\boxed{\sqrt{}}$ $\boxed{a/b}$ $\boxed{1}$ $\boxed{-}$ $\boxed{\cos}$ $\boxed{X/\theta/T/n}$ $\boxed{\blacktriangledown}$ $\boxed{2}$ $\boxed{ENTER}$.

5. Sin ($\frac{X}{2}$) is positive for $0 < x < 2\pi$. Enter this range by pressing $\boxed{WINDOW}$ $\boxed{0}$ $\boxed{ENTER}$ $\boxed{2}$ $\boxed{2ndF}$ $\boxed{\pi}$ $\boxed{ENTER}$ $\boxed{2ndF}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{ENTER}$ $\boxed{(-)}$ $\boxed{2}$ $\boxed{ENTER}$ $\boxed{2}$ $\boxed{ENTER}$ $\boxed{1}$ $\boxed{ENTER}$.

6. Check to see if they result in the same graph by pressing $\boxed{GRAPH}$.
    Sketch what you see in the box below:
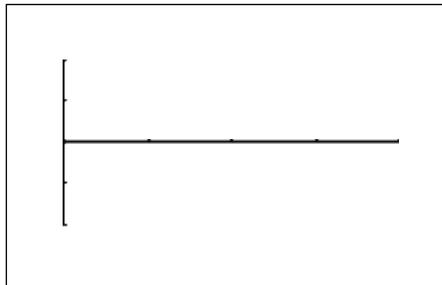
3.4

GRAPHICAL VERIFICATION OF FORMULAS AND IDENTITIES

Activity 2   Verify the formula from Activity 1 for intervals where sine is negative:

1. Press $\boxed{\text{Y=}}$ $\boxed{\blacktriangledown}$ to access the Y2 prompt.

2. Press $\boxed{(-)}$ to insert a negative in front of the radical.

3. Sin ($\frac{x}{2}$) will be negative for $2\pi < x < 4\pi$. Enter this range by pressing $\boxed{\text{WINDOW}}$ $\boxed{2}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$ $\boxed{4}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$.

4. Check to see if they result in the same graph by pressing $\boxed{\text{GRAPH}}$.

5. Sketch the graph in the box below.
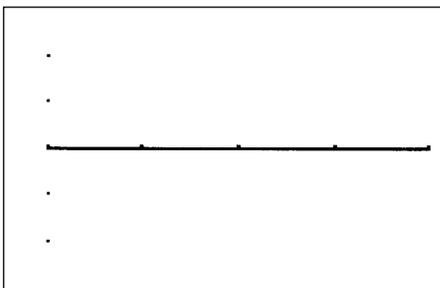
3.5

GRAPHICAL VERIFICATION OF FORMULAS AND IDENTITIES

Activity 3   Verify the identity $\tan^2\theta + 1 = \sec^2\theta$:

1. Access and clear the Y1 prompt.

2. Enter the $\tan^2 x + 1$ for Y1.

3. Enter the $\sec^2 x$ for Y2.

4. Set your viewing window.

5. Check to see if they result in the same graph.

6. Sketch the graph in the box below.
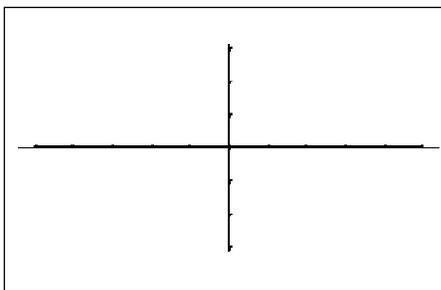
Activity 4   Verify the following formulas and identities using the steps provided in Activities 1 through 3:

1. $\cos 2\theta = 2\cos^2\theta - 1$

2. $\csc^2\theta - \cot^2\theta = 1$

3. $\tan\theta\csc\theta = \sec\theta$

4.1

AMPLITUDE

Investigate changes in amplitude with the SHIFT/CHANGE feature.

1. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SHIFT/CHANGE}}$, touch **B CHANGE**, and double touch **5 Y=sin X**.

2. Double touch the screen at Y = 3, above the box with the arrows. Notice the new function Y = 3 sin X is displayed.

3. Press $\boxed{\text{ENTER}}$ to place the new function in the list.

4. Double touch the screen at **Y = ⁻2**, below the box with the arrows. Notice the new function Y = ⁻2 sin X is displayed.

5. Press $\boxed{\text{ENTER}}$ to place the new function in the list.

6. Touch the top of the right side of the screen and the list of three functions will be displayed.

7. The function Y = sin X is highlighted on the right side of the screen, and the graph is darkened on the left side of the screen.

8. Press $\boxed{\blacktriangledown}$ to move from one graph to the next. Observe and note the changes in sin X when you change the amplitude.

_____

_____

_____

4.2

PERIOD AND PHASE SHIFT

Investigate the change in a trigonometric function by multiplying the argument θ by 2.

1. Press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear Y1 of an old expression.

2. Make sure the calculator is in radian mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu.

3. Now, enter sin x for Y1 by pressing $\boxed{\text{sin}}$ $\boxed{\text{X/θ/T/}n}$ $\boxed{\text{ENTER}}$ and the sin (2*x*) for Y2 by pressing $\boxed{\text{CL}}$ $\boxed{\text{sin}}$ $\boxed{(}$ $\boxed{2}$ $\boxed{\text{X/θ/T/}n}$ $\boxed{)}$ $\boxed{\text{ENTER}}$.

4. Set your viewing window by pressing $\boxed{\text{ZOOM}}$, touching **E TRIG**, and double touching **1 sin *x***. Note the change in the second graph versus the first graph. Engage the trace to show the period has changed from 2π to π.

Investigate the change in a trigonometric function by subtracting $\frac{3\pi}{2}$ to the argument θ.

1. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SHIFT/CHANGE}}$, touch **A SHIFT**, and double touch **6 Y = sin X**. Double touch the screen at X = 3π/2 to the right of the box with the arrows. Notice the new function Y = sin (X - 3π/2) is displayed. Press $\boxed{\text{ENTER}}$ to place the new function in the list.

2. Double touch the screen at Y = ⁻2, below the box with the arrows. Notice the new function Y = sin (X – 3π/2) ⁻2 is displayed. Press $\boxed{\text{ENTER}}$ to place the new function in the list. Touch the top of the right side of the screen and the list of three functions will be displayed. The function Y = sin X is highlighted on the right side of the screen, and the graph is darkened on the left side of the screen. Press $\boxed{\blacktriangledown}$ to move from one graph to the next. Observe and note the changes in sin X when you change the phase shift.

4.3

AMPLITUDE, PERIOD, AND PHASE SHIFT

Activities   **Enter the first graph for Y1 with the second graph for Y2 and note the change(s).**

COMPARE   CHANGE(S)

1. cos x with 2cos x   _____

2. cos x with $(\frac{1}{2})$cos x _____

3. cos x with cos 2x _____

4. cos x with cos ($\frac{x}{2}$)   _____

5. cos x with cos($x - \frac{\pi}{6}$)   _____

6. cos x with cos($x + \frac{\pi}{6}$)   _____

7. cos x with 3cos($x + \frac{\pi}{6}$)   _____

8. tan x with 2tan($\frac{x}{3}$)   _____

9. cot x with cot($\frac{x}{2}$ – $\frac{\pi}{3}$)   _____

10. sec x with 2sec(3x)   _____

11. csc x with $(\frac{1}{2})$csc($x - \frac{\pi}{6}$)   _____

5.1

INVERSE TRIGONOMETRIC FUNCTIONS

To compute the inverse sine function in degrees:

1. Turn the calculator on and press ▦ CL to enter computation mode.

2. Change to degree mode by pressing 2ndF SET UP , touching **B DRG**, and double touching **1 Deg**. Press 2ndF QUIT to exit the menu.

3. Find $\sin^{-1}$ (.5) by pressing 2ndF $\sin^{-1}$ • 5 ENTER . The answer of 30° is shown below.

```
sin⁻¹ .5
                    30
```

To graph the inverse sine function in degrees:

1. Press Y= CL to access and clear the Y1 expression.

2. Press ENTER CL to clear additional Y expressions.

3. Enter $\sin^{-1}$ *x* for Y1 by pressing 2ndF $\sin^{-1}$ X/θ/T/*n* ENTER .

4. Set your viewing window by pressing ZOOM , touching **E TRIG**, and double touching **4 $\sin^{-1}$ *x***.
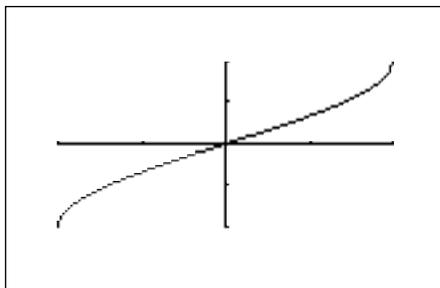
5.2

INVERSE TRIGONOMETRIC FUNCTIONS

To compute the inverse sine function in radians:

1. Turn the calculator on and press ⊞⊟⊠⊡ CL to access and clear the home screen.

2. Change to radian mode by pressing 2ndF SET UP , touching **B DRG**, double touching **2 Rad**. Press 2ndF QUIT to exit the menu.

3. Find sin⁻¹ (.4), press 2ndF sin⁻¹ • 4 ENTER . The answer of .4115 radians is shown below. To find the result in terms of π press ÷ 2ndF π ENTER .

```
sin⁻¹ .4
                .411516846
Ans/π
                .13098988
```
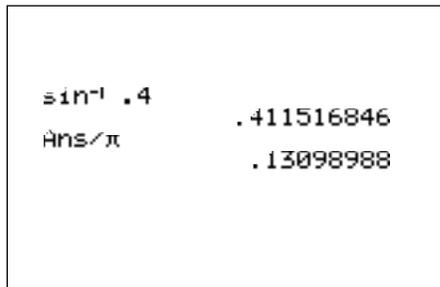
To graph the inverse cosine function in radians:

1. Press Y= CL to access and clear the Y1 prompt.

2. Enter cos ⁻¹ x for Y1 by pressing 2ndF cos⁻¹ X/θ/T/n ENTER .

3. Set your viewing window by pressing ZOOM , touching **E TRIG**, and double touching **5 cos⁻¹ x**.
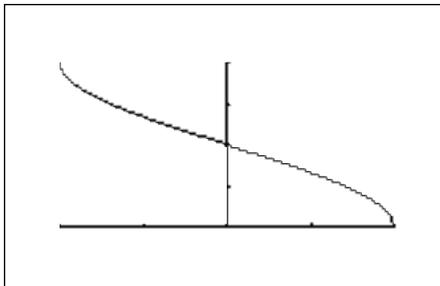
5.3

INVERSE TRIGONOMETRIC FUNCTIONS

Activities

Compute the following.

1. $\sin^{-1} .8 =$ _____ degrees

2. $\cos^{-1} (^-.6) =$ _____ degrees

3. $\tan^{-1} 2 =$ _____ degrees

4. $\sec^{-1} (^-1.3) =$ _____ radians

5. $\csc^{-1} (1.7) =$ _____ radians

6. $\cot^{-1} (^-1) =$ _____ radians

Graph the following.

7. $y = \tan^{-1} x$

8. $y = \cot^{-1} x$

6.1

SOLVING TRIGONOMETRIC EQUATIONS

To solve a trigonometric equation graphically using the zoom feature:

1. Turn the calculator on and press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear Y1 of an old expression. Set the calculator to degree mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu.

2. To find the solutions of $3\sin^2 x - 4\sin x + 1 = 0$, press $\boxed{3}$ $\boxed{(}$ $\boxed{\sin}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{-}$ $\boxed{4}$ $\boxed{\sin}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{+}$ $\boxed{1}$ $\boxed{\text{ENTER}}$.

3. Set your viewing window to the principal values for sine by pressing $\boxed{\text{WINDOW}}$ $\boxed{(-)}$ $\boxed{9}$ $\boxed{0}$ $\boxed{\text{ENTER}}$ $\boxed{9}$ $\boxed{0}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{0}$ $\boxed{\text{ENTER}}$ $\boxed{(-)}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to view the graph.

4. To save this original graph and viewing window, press $\boxed{\text{ZOOM}}$, touch **G STO**, and double touch **1 StoWin**.

5. Set the zoom factors by pressing $\boxed{\text{ZOOM}}$, touching **B FACTOR**, pressing $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to exit the menu.

6. To find the left intercept press $\boxed{\text{TRACE}}$ and move the cursor near this intercept by continuing to press the $\boxed{\blacktriangleright}$ key.

7. Now, press $\boxed{\text{ZOOM}}$, touch **A ZOOM**, and double touch **3 In** to zoom in on the left intercept and solution of this equation. Press $\boxed{\text{TRACE}}$ and $\boxed{\blacktriangleleft}$ to trace near and approximate the intercept.

8. Return to the original screen by pressing $\boxed{\text{ZOOM}}$, touching **H RCL**, and double touching **1 RclWin**. Trace to find the right intercept.

6.2

SOLVING TRIGONOMETRIC EQUATIONS

To solve a trigonometric equation graphically using the CALC feature:

1. Press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear Y1. Set the calculator to radian mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the menu.

2. To find the solutions of $2\sin^2 x - \cos x - 1 = 0$, press $\boxed{2}$ $\boxed{(}$ $\boxed{\sin}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{-}$ $\boxed{\cos}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{-}$ $\boxed{1}$ $\boxed{\text{ENTER}}$.

3. Set your viewing window by pressing $\boxed{\text{WINDOW}}$ $\boxed{(-)}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{\text{ENTER}}$ $\boxed{(-)}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to view the graph.

4. To "calculate" an intercept (solution), press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **5 X_Incpt**. You can calculate the other two intercepts by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touching **5 X_Incpt** again.

To solve an equation with functions on both the left and right sides:

1. Press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to access and clear Y1 of an old expression. To find the solutions of $2\tan x \sin x + 2\sin x = \tan x + 1$, you will enter the left side as Y1, by pressing $\boxed{2}$ $\boxed{\tan}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\sin}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\sin}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{ENTER}}$.

2. Enter the right side as Y2, by pressing $\boxed{\tan}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{+}$ $\boxed{1}$ $\boxed{\text{ENTER}}$.

3. Set your viewing window by pressing $\boxed{\text{WINDOW}}$ $\boxed{(-)}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{\text{ENTER}}$ $\boxed{(-)}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to view the graph.

4. There appears to be four solutions to the equation. To "calculate" the intersections (solutions), press $\boxed{\text{TRACE}}$, move the tracer right until it appears on the screen. Next, press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **2 Intsct**. Do this several times.

6.3

SOLVING TRIGONOMETRIC EQUATIONS

Activity 1   Solve the following with the zoom feature.

1. $2\sin^2 x + 3\sin x - 2 = 0$ _____ degrees

2. $2\sin x + \sqrt{3} = 0$ _____ degrees

3. $4\cos^2 x = 1$ _____ degrees

4. $\sin 2x = \cos x$ _____ radians

5. $\tan x + \cot x = 2$ _____ radians

Activity 2   Solve the following with the CALC feature.

1. $\sin x + \cos x = 0$ _____ degrees

2. $\sin^2 x = \cos x - 1$ _____ degrees

3. $3\cos 2x - 5\cos x = 1$ _____ radians

4. $\sqrt{3}\cot x \sin x + 2\cos^2 x = 0$ _____ radians

5. $4\tan x + \sin 2x = 0$ _____ radians

7.1

APPLICATIONS WITH RIGHT TRIANGLES

Steps for using the calculator to solving applications with right triangles

1. Turn the calculator on and set it to degree mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up menu.

2. Given a right triangle with A = 3 and C = 6, find B and θ using the SOLVER. To access the SOLVER, press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, and press $\boxed{\text{CL}}$ to delete any equation that may appear on the screen.

3. Type in the equation $\sin \theta = \dfrac{A}{C}$ with the keystrokes $\boxed{\text{sin}}$ $\boxed{\text{ALPHA}}$ $\boxed{\theta}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{C}}$.

4. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation** to choose the Equation solver. Press $\boxed{\text{ENTER}}$ and see the list of variables.

5. Press $\boxed{\blacktriangledown}$ to move the blinking cursor over the A and press $\boxed{3}$ $\boxed{\text{ENTER}}$. Type in the value of C by pressing $\boxed{6}$ $\boxed{\text{ENTER}}$. Solve for θ by pressing $\boxed{\blacktriangle}$ $\boxed{\blacktriangle}$ $\boxed{\text{2nd}}$ $\boxed{\text{EXE}}$.

6. To use the SOLVER to find B, edit the current equation to the new equation $\cos \theta = \dfrac{B}{C}$. Do this by pressing $\boxed{\text{CL}}$ $\boxed{\text{CL}}$ to display the current equation $\sin \theta = \dfrac{A}{C}$. With the cursor over the s in sin, press $\boxed{\text{cos}}$ to insert cos. Press $\boxed{\text{DEL}}$ to delete sin. Press $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleright}$ to move the cursor over the A in the numerator of the fraction. Press $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ to insert B. Press $\boxed{\text{DEL}}$ to delete A.

7. Press $\boxed{\text{ENTER}}$ to see the variable list. The value for θ and C will remain as before. Press $\boxed{\blacktriangledown}$ to position the cursor over the B and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find B.

7.2

APPLICATIONS WITH RIGHT TRIANGLES

A rectangle has length 16 cm and width 8.6 cm. Find the degree measure of angle A. Determine the length of the diagonal of the rectangle.
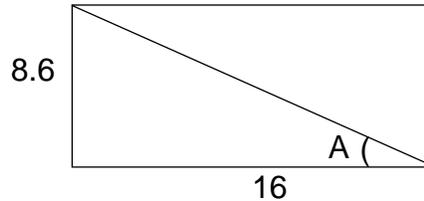
1. Press ⊞⊟⊠⊞ 2ndF SOLVER to access the SOLVER, and press CL to delete the equation that is on the screen.

2. Enter the equation $\tan \theta = \dfrac{A}{B}$ by pressing tan ALPHA θ ALPHA = a/b ALPHA A ▶ ALPHA B .

3. Press ENTER to view the variable list.

4. Enter 8.6 for A and 16 for B. (The value of θ will be listed as the one that was used in the previously entered equation.)

5. Position the cursor over θ and press 2ndF EXE to find θ.

6. To determine the length of the diagonal of the rectangle, return to the equation by pressing CL CL and edit the current equation to read $\sin \theta = A/C$.

7. Press ENTER to see the variable list. The value of θ should be as before, so enter 8.6 for A and solve for C.

7.3

APPLICATIONS WITH RIGHT TRIANGLES

Use the solver with sine, cosine, and tangent functions.

1. Let ABC be a right triangle with AB=32, BC=12.5, and right angle at C.
   Find the degree measure of the acute angle at A and side AC.
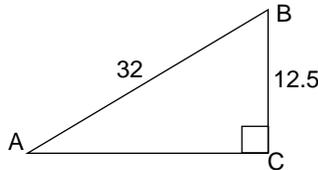
2. A wheelchair ramp will be built so that it rises to a height of 3 feet above
   level ground. If the end of the base of the ramp is 12 feet from the building,
   how long is the ramp? What is the degree measure of the angle at the end of
   the ramp and level ground?
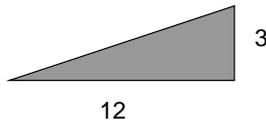
3. A guy wire for a television antenna makes an angle of 60° with the mast.
   If the wire is attached to the antenna 50 feet from the base of the antenna,
   use the secant function to determine the length of the wire.
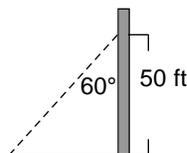
4. John is lying on the beach 500 feet from his hotel and looking at Paula who is
   standing on a balcony. If the balcony is 450 feet above ground level, use the
   cotangent function to determine the angle between the beach and John's line
   of sight.
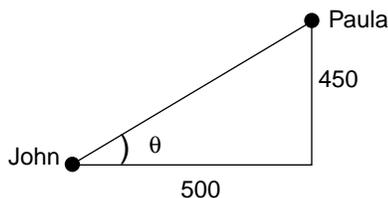
8.1

LAW OF SINES

Use the Graphic Solver with the Law of Sines.

1. Set the calculator to degrees by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up screen.

2. To access the SOLVER, press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$ $\boxed{\text{CL}}$ Enter sin $\frac{A}{X}$ = sin $\frac{B}{Y}$ by pressing $\boxed{\text{a/b}}$ $\boxed{\text{sin}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{\text{sin}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$.

3. Store it into memory by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touching **C SAVE**, pressing $\boxed{\text{ENTER}}$, and typing in LAWSINES. Press $\boxed{\text{ENTER}}$ to store the formula.

4. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, double touch **3 Graphic** to choose the Graphic Solver. Press $\boxed{\text{ENTER}}$ for the list of variables.

5. With the blinking cursor over the A, type in 95 and press $\boxed{\text{ENTER}}$. Next, type 15 for *x*, the length of the side opposite the 95°, and press $\boxed{\text{ENTER}}$. Enter 48 for B in the same manner. Solve for *y* by positioning the blinking cursor over the *y* and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$.

6. You must enter the range values for the unknown (from BEGIN to END) within which you expect to find the solution. Let the BEGIN = 5 by pressing $\boxed{5}$ $\boxed{\text{ENTER}}$. Enter a value for END that you do not expect *y* to exceed, say 25, by pressing $\boxed{2}$ $\boxed{5}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ and an autoscaled plot will be drawn showing graphs of the left and right sides of the current equation.

7. The calculator will now display a blinking box in the upper right-hand corner of the viewing screen while it looks for the intersection of the two graphs. The solution is displayed at the bottom of the screen.

8.2

LAW OF COSINES

Use the Newton Solver with the Law of Cosines.

1. Access the SOLVER with $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$ (or press $\boxed{\text{CL}}$ twice if you are still in the SOLVER). Press $\boxed{\text{CL}}$ to delete the current equation. Return to the Equation Solver by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation**.

2. Next enter the Law of Cosines by pressing $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{x^2}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{2}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{\times}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{\times}$ $\boxed{\text{cos}}$ $\boxed{\text{ALPHA}}$ $\boxed{\theta}$.

3. Store this equation in the calculator.

4. Given a triangle in which two sides of lengths 13 cm and 8 cm enclose a 50° angle. Find the length of the third side.

5. Press $\boxed{\text{ENTER}}$ to view the list of variables. Press $\boxed{\blacktriangledown}$ to position the cursor on X and enter 13. Next, enter **8** for Y and 50 for θ. Press $\boxed{\blacktriangle}$ to move the cursor over Z and press $\boxed{0}$ $\boxed{\text{ENTER}}$.

6. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to solve for Z. Notice that the SOLVER has automatically changed to the Newton Solver. You should supply an initial estimate of the solution in the START position (0 is the default value) and a step size STEP (.001 is the default value).

7. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the solution. (If you receive an error message, try changing the START value or increasing the STEP value.)

8. Notice the left and right sides of the equation are displayed as equal values, and the L – R = 0.

8.3

# LAW OF SINES AND LAW OF COSINES

Use the SOLVER with the Law of Sines.

1. A 50 foot tree has been damaged
   in a storm and is inclined at an
   angle of $\theta°$ (from the vertical)
   directly away from the sun. Find
   if the tree casts a shadow 40 feet
   long on level ground when the
   angle of elevation of the sun is 60°.

B°

50 ft

$\theta°$

60°

40 ft
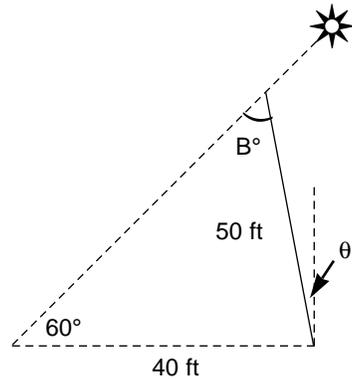
Use the SOLVER with the Law of Cosines.

2. A bridge is to be built between
   points B and C across a lake. A
   surveyor using a transit determines
   the measure of the angle at point A
   to be 82°. The distance between
   points A and B and between points
   A and C is known to be 125 yards.
   Find the distance across the lake
   that the bridge will span.

B

A 82°

d

125

C
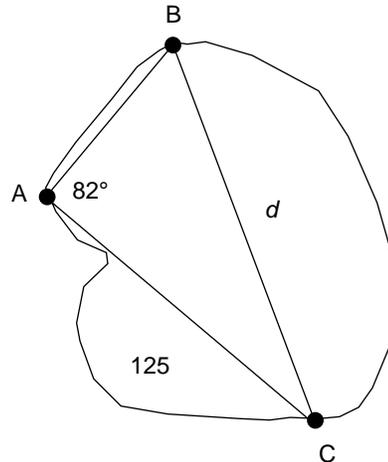
3. Two adjacent sides of a parallelogram
   are of length 8 and 12. The length of
   the shorter diagonal of the parallelogram
   is 10.5. Find the angle opposite the
   shorter diagonal.

8

10.5

θ

12

9.1

AREA OF A TRIANGLE

1. Set the calculator to operate in degrees with $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **B DRG**, and double touch **1 Deg**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up screen.

2. Access the SOLVER by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, and press $\boxed{\text{CL}}$ to delete any equation on the screen.

3. Type in the equation A = $\frac{1}{2}$ B•H with the keystrokes: $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{1}$ $\boxed{\blacktriangleright}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\times}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{H}}$

4. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation** to choose the Equation solver. Press $\boxed{\text{ENTER}}$ to see the list of variables for your equation.

5. Find the area of a triangular garden plot with base = 4 meters and height = 8.35 meters, use $\boxed{\blacktriangledown}$ to move the blinking cursor over B and type in 4. Type in 8.35 for H and press $\boxed{\text{ENTER}}$. Press $\boxed{\blacktriangle}$ twice to move the prompt to the A variable. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area, 16.7 square meters.

6. Save the formula by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$ and touching **C SAVE**. Enter a name and press $\boxed{\text{ENTER}}$.

7. Enter A = $\frac{1}{2}$ B•C•sin θ by pressing $\boxed{\text{CL}}$ a couple times to return to and delete the current equation. Type in the equation with the keystrokes: $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{a/b}}$ $\boxed{1}$ $\boxed{\blacktriangleright}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\times}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{C}}$ $\boxed{\times}$ $\boxed{\text{sin}}$ $\boxed{\text{ALPHA}}$ $\boxed{θ}$.

8. Consider the triangle where sides of lengths 4.8 and 5.73 enclose a 47° angle. Press $\boxed{\text{ENTER}}$ to access the variable list, enter 5.73 for B, 4.8 for C (or vice-versa), and 47 for θ. Move the cursor to the A variable. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to solve for the area.

9.2

AREA OF A TRIANGLE

1. Continue with the formula from *Blackline Master 9.1*. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **3 Graphic** to choose the Graphic solver.

2. To start the problem from the beginning, set A to 0. Move the cursor to the A variable and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to see the Graphic solver variable range screen.

3. Since area is always nonnegative, 0 could be used for BEGIN. Enter END a value that you do not expect area to exceed, say 25. (A little bit of estimation helps here! If your value for END is too small, increase it and try again.)

4. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ and an autoscaled graph of the two sides of the equation is drawn and the solution is obtained.

5. Enter Heron's formula, $A = \sqrt{[S(S\text{-}X)(S\text{-}Y)(S\text{-}Z)]}$, press $\boxed{\text{CL}}$, and type the following: $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{\text{2ndF}}$ $\boxed{\sqrt{}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{-}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{X}}$ $\boxed{)}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{-}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{)}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{S}}$ $\boxed{-}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{)}$.

6. Note that the square root symbol extends over the entire right hand side of the equation, automatically grouping the expression under it.

7. You could enter the semiperimeter, $S = \frac{1}{2}(X + Y + Z)$, in a separate formula in the SOLVER, but it is probably easier to find the value of S in the computation mode and then use Heron's formula in the SOLVER. Press $\boxed{\text{ENTER}}$ to view the variable list.

8. Use Heron's formula to solve some area problems.

9.3

AREA OF A TRIANGLE

Use the solver to find the area of triangle.

1. Two legs of a right triangle (neither of which is the hypotenuse) are of length 25 and 38. Find the area of the triangle.

2. Triangle ABC has two sides of lengths $13\sqrt{3}$ cm and 18 cm that enclose angle *q*. The area of triangle ABC is 132.95 square cm. Find the height *h* of triangle ABC. Find the measure of angle θ.

```
                        B
                       /|\
                    18/ | \
                     /  |h \
                    /   |   \
                 θ /    |    \
              A  /_)_____|_____\ C
                    13√3
```
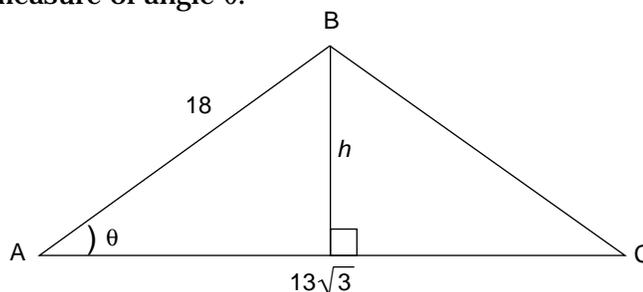
3. A wooded lot has the shape of ∆STR in the figure below. The highway department plans to purchase the region shown as ∆MOR for frontage on a proposed interstate highway. A survey of the region shows the distance from point M to point O is 60 feet, the distance from R to M is 85 feet, and the angle enclosed by these two lengths is 65°. The deed for the lot indicates that the distance from S to T is 170 feet and the distance from S to R is 245 feet. The owner of the lot has determined that the area of the four-sided region remaining after the highway department purchase is 16,244 square feet. Find the area of the region the highway department plans to purchase. Find the measure of angle at the back corner of the wooded lot.

```
                         T
                        /|
                170 ft./ |
                      /  |
                   θ°/   |
                  S /___/|
                       / O
                      /  /
                     M  /65°
                        /
                       R
```
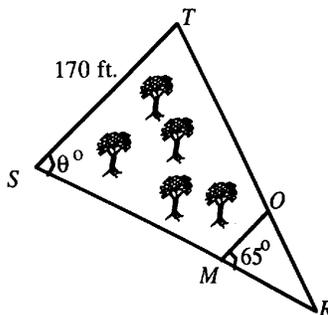
9.4

AREA OF A TRIANGLE
Use the solver with Heron's formula.

4. A spider web in the shape of a regular octagon is such that each of the eight triangular regions shown in the figure has the same shape and size as the shaded region. If the distance from the center of the web to point R is 8 inches and equals the distance from the center of the web to point S, find the total area enclosed by the web. Find the perimeter of the web.
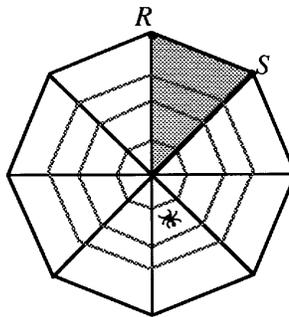
5. Yum-Yum Burgers Corporation is considering purchasing a corner lot in town for a drive-through hamburger restaurant. They have decided to by the land if the price is $9000 or less. If the lot is triangular in shape with sides of lengths 100 feet, 50 feet, 72 feet and the owner is asking $5 per square foot, will Yum-Yum Burgers purchase the lot?

6. Jim is lost in the forest. The police have determined that they should search the shaded region in the figure below. If each unit on the grid represents one mile, what is the area of region that the police will search?
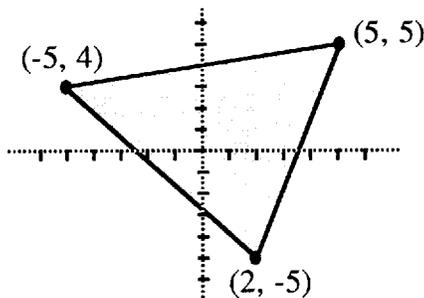
10.1

AREA OF A CIRCULAR SECTOR

A lawn sprinkler is set so that the region of the lawn that is watered is approximately a sector of a circle of radius 6 feet. If the central angle of the sector is $\frac{\pi}{5}$ radians, find the area of the region that is watered by the sprinkler.

1. Set the calculator to radian mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **2 Rad**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the set up screen.

2. Access the SOLVER by pressing $\boxed{\boxplus}$ $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$. Press $\boxed{\text{CL}}$ to delete any equation that appears on the screen.

3. Type in the equation A= $\frac{1}{2}$ R²θ by pressing $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ALPHA}}$ $\boxed{=}$ $\boxed{1}$ $\boxed{\text{a/b}}$ $\boxed{2}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{R}}$ $\boxed{x^2}$ $\boxed{\text{ALPHA}}$ $\boxed{\theta}$.

4. Use the Equation solver by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **1 Equation**. Press $\boxed{\text{ENTER}}$ and see the list of variables.

5. Use $\boxed{\blacktriangledown}$ to place the cursor over R, type in 6, and press $\boxed{\text{ENTER}}$. Type in θ by pressing $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{\text{ENTER}}$. Press $\boxed{\blacktriangle}$ twice to place the cursor on the A variable. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area in square feet.

6. Use the Newton solver by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **2 Newton**. Reset A to 0 to start the problem from the beginning. The values of R and O are correct for this problem, so use $\boxed{\blacktriangle}$ to place the cursor over the A variable and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to view the Newton solver screen.

7. Since area is never negative, you could set START = 0. Leave the STEP size as .001 and remember that you should change it to a slightly larger value if an error message results. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area in square feet.

10.2

AREA OF A CIRCULAR SECTOR

1. Using the problem from *Blackline Master 10.1*, find the area using the Graphic Solver.

2. Press $\boxed{\text{2ndF}}$ $\boxed{\text{SOLVER}}$, touch **A METHOD**, and double touch **3 Graphic**.

3. Reset A to 0 to start the problem from the beginning.

4. Place the cursor on the A variable and press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$.

5. Recall that BEGIN and END are the lower and upper values between which you feel the solution will lie. Enter a 20 for END so that the intersection of the two graphs is visible on the screen.

```
Graphic solver
variable range

BEGIN=0

END=20
```

6. Press $\boxed{\text{2ndF}}$ $\boxed{\text{EXE}}$ to find the area.

```
A=11.30973356
```

10.3

AREA OF A CIRCULAR SECTOR

Use the Equation or Newton solver to find the area of a circular sector.

1. Find the area of a circular sector subtended by a central angle of 0.6π radians in a circle of radius 5 yards.

2. A sector of a circle has an area equal to 36 square inches. If the arc of the sector subtends an angle of 55° at the center of the circle, find the radius of the circle.

3. A large pizza in the shape of a circle has a diameter of 20 inches. The pizza is cut into 8 slices of equal area. What is the area of each piece of pizza?

Use the Graphic solver to find the area of a circular sector.

4. A circular clock has the tip of the hour hand touching the rim of the clock. What is the area of the sector formed when the hour hand moves from 1 o'clock to 5 o'clock?

5. A cherry cheesecake is in the shape of a circle. If the diameter of the pan holding the cheesecake is 9 inches, and the cheesecake is cut into 10 equal pieces, find the area covered by the cherry topping on one piece of the cheesecake.

KEYPAD FOR
THE SHARP EL-9600 GRAPHING CALCULATOR
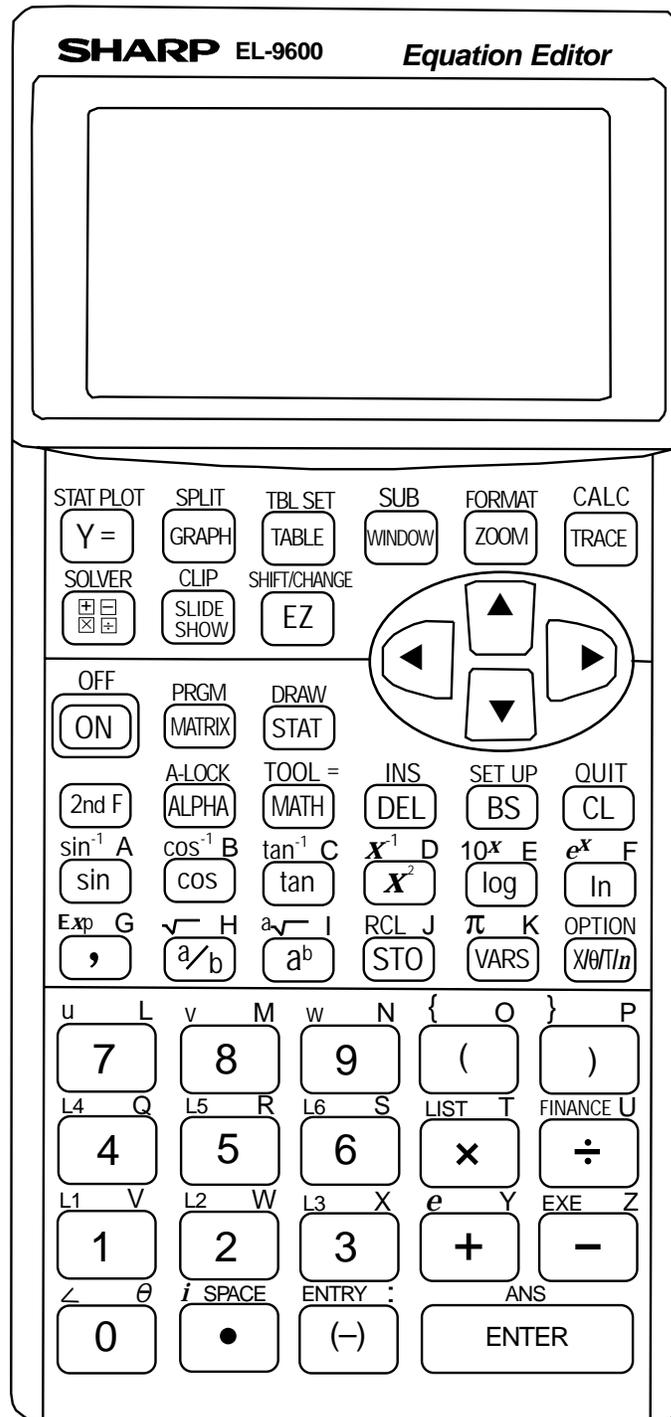
SOLUTIONS TO SELECTED ACTIVITIES

2. EVALUATING TRIGONOMETRIC FUNCTIONS
ADDITIONAL PROBLEMS
1. .8910
2. .7813
3. 2
4. .9563
5. .4816
6. -1.4142

BLACKLINE MASTERS 2.3-2.4

ACTIVITY 1	ACTIVITY 2
1. .9848	7. -.5878
2. .9848	8. -.8660
3. 2.7475	9. 1
4. 1.2208	10. -1.4142
5. 1.0785	11. -2
6. 1.1918	12. -2.0765
ACTIVITY 3	ACTIVITY 4
13. 3090	19. .5878
14. .8387	20. -.9010
15. 1.0355	21. -1
16. 1.0038	22. 2.9238
17. -1.2208	23. 2.6131
18. .7002	24. .1438

3. GRAPHICAL VERIFICATION OF FORMULAS OR IDENTITIES
ADDITIONAL PROBLEMS
1. Identity
2. Identity
3. Identity

BLACKLINE MASTER 3.3
Identity

BLACKLINE MASTER 3.4
Identity

BLACKLINE MASTER 3.5

ACTIVITY 3	ACTIVITY 4
Identity	1. Identity
	2. Identity
	3. Identity

4. AMPLITUDE, PERIOD AND PHASE SHIFT
ADDITIONAL PROBLEMS
1. flipped over, rising faster
2. period is 3π
3. shifted right $\frac{\pi}{2}$
4. shifted left $\frac{\pi}{2}$ and up 2

BLACKLINE MASTER 4.1
3 sin x has an amplitude of 3, -2sin x is flipped over with an amplitude of 2

BLACKLINE MASTER 4.2
The function has moved 3 $\frac{\pi}{2}$ to the right, and down 2

BLACKLINE MASTER 4.3
1. larger amplitude
2. smaller amplitude
3. smaller period
4. larger period
5. moved right $\frac{\pi}{6}$
6. moved left $\frac{\pi}{6}$
7. larger amplitude, moved left $\frac{\pi}{12}$, smaller period
8. pinched in, larger period
9. larger period, moved right $\frac{2\pi}{3}$
10. moved farther from x-axis, smaller period
11. moved closer to x-axis, moved right $\frac{\pi}{6}$

5. INVERSE TRIGONOMETRIC FUNCTIONS
ADDITIONAL PROBLEMS
1. -44.427°
2. 60°
3. -63.4349°
4. -30°
5. .2014
6. 2.2143
7. 1.2490
8. .3218
9. .8411

BLACKLINE MASTER 5.3
1. 53.1301°
2. 126.8698°
3. 63.4349°
4. 2.4484
5. .6289
6. -.7854

6. SOLVING TRIGONOMETRIC EQUATIONS
ADDITIONAL PROBLEMS
1. -60° and 60°
2. 90°
3. -90, 30°, 150°
4. 3.1416

5. 0, 3.1416
6. .5236, 2.618

BLACKLINE MASTER 6.1
7. 19.4°
8. 90°

BLACKLINE MASTER 6.2
ACTIVITY 1
1.0472, 3.1416, -1.0472

ACTIVITY 2
2.3562, 2.618, -.7854, .5236

BLACKLINE MASTER 6.3
ACTIVITY 1
1. 30°, 150°
2. -120°, -60°
3. 60°, 120°
4. .5236, 1.5708, 2.6180
5. .7854

ACTIVITY 2
1. -45°, 135°
2. 0°
3. -2.0944, 2.0944,
4. 1.5708, 2.6180, 3.6652, 4.7124
5. 0

7 APPLICATIONS WITH RIGHT TRIANGLES
ADDITIONAL PROBLEMS
1. 5.945
2. 37.859
3. 36.87°
4. 3.849
5. No, one possible explanation is that sin > 1 which is not possible
6. 216.306

BLACKLINE MASTER 7.1
5. 30°
7. 5.196

BLACKLINE MASTER 7.2
5. 28.258°
7. 18.165

BLACKLINE MASTER 7.3
1. 22.993°, 29.458
2. 12.369, 14.036°
3. 100 feet
4. 41.987°

8 LAW OF SINES AND LAW OF COSINES
ADDITIONAL PROBLEMS
1. 1447.761
2. 766.986
3. 502.341
4. 1521.317

BLACKLINE MASTER 8.1
7. 11.1898

BLACKLINE MASTER 8.2
7. 9.9649

BLACKLINE MASTER 8.3
1. 13.854°
2. 200 yards
3. 59.395°

9 AREA OF A TRIANGLE
ADDITIONAL PROBLEMS
1. 30.311
2. 1.5
3. 190.84
4. 12
5. no intersection, sin > 1 impossible

BLACKLINE MASTER 9.1
8. 10.06

BLACKLINE MASTER 9.3
1. 475
2. 11.809, 41°
3. 2311.085, 63°

BLACKLINE MASTER 9.4
4. 181
5. 8521.68 < $9000
6. 48.5

10   AREA OF A CIRCULAR SECTOR
ADDITIONAL PROBLEMS
1. 42.412
2. 6.981
3. 2.611 radians or 149.606°

BLACKLINE MASTER 10.1
5. 11.3097
7. 11.3097

BLACKLINE MASTER 10.3
1. 23.562
2. 8.661
3. 39.27
4. $\pi r^2/3$
5. 6.362

TEACHING NOTES

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