

APPLYING

# PRE-ALGEBRA/ALGEBRA



SHARP

USING THE  
**EL-9600**  
*Graphing Calculator*

DAVID P. LAWRENCE

Applying  
**PRE-ALGEBRA/ALGEBRA**  
using the  
**SHARP EL-9600**  
**GRAPHING CALCULATOR**

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This Teaching Resource has been developed specifically for use with the Sharp EL-9600 graphing calculator. The goal for preparing this book was to provide mathematics educators with quality teaching materials that utilize the unique features of the Sharp graphing calculator.

This book, along with the Sharp graphing calculator, offers you and your students 10 classroom-tested, topic-specific lessons that build skills. Each lesson includes *Introducing the Topic*, *Calculator Operations*, *Method of Teaching*, explanations for *Using Blackline Masters*, *For Discussion*, and *Additional Problems* to solve. Conveniently located in the back of the book are 38 reproducible *Blackline Masters*. You'll find them ideal for creating handouts, overhead transparencies, or to use as student activity worksheets for extra practice. *Solutions to the Activities* are also included.

We hope you enjoy using this resource book and the Sharp EL-9600 graphing calculator in your classroom.

*Other books are also available:*

Applying STATISTICS using the SHARP EL-9600 Graphing Calculator  
Applying TRIGONOMETRY using the SHARP EL-9600 Graphing Calculator  
Applying PRE-CALCULUS and CALCULUS using the SHARP EL-9600 Graphing Calculator  
Graphing Calculators: Quick & Easy! The SHARP EL-9600

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*Dedicated to my grandparents, Harold and Ruby Sharp*

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# Chapter one

## LINEAR EQUATIONS

### Introducing the Topic

In this chapter, you and your students will learn how to graph linear equations and study their characteristics. These characteristics include slope,  $y$ -intercept, perpendicular, and parallel.

A linear equation of  $y$  in terms of  $x$  can be expressed by the slope-intercept form  $y = m \cdot x + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept. We call this equation a linear equation, since its graph is a straight line. Equations where the exponents on the  $x$  and  $y$  are 1 (implied) are considered linear equations. In graphing linear equations on the calculator, we will let the  $x$ -variable be represented by the horizontal axis and let  $y$  be represented by the vertical axis.

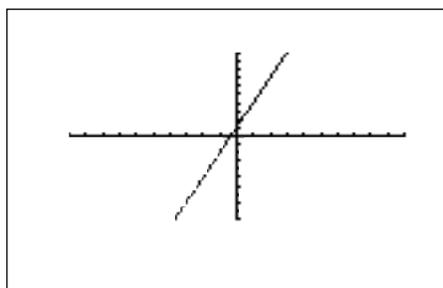
### Calculator Operations

Turn the calculator on and press  $\boxed{Y=}$ . A set of  $Y$  prompts will appear on the viewing screen. Press  $\boxed{CL}$  to remove an old  $Y$  expression. Move up or down the set of  $Y$  prompts using the arrow keys, or simply touch the screen twice at the  $Y$  prompt desired.

The calculator should be set up with rectangular coordinates and in equation editor mode. To complete this setup, press  $\boxed{2\text{ndF}} \boxed{\text{SET UP}} \boxed{\text{E}}$  (COORD) *(Note that the alphabet letters are written above and to the right of the access key and can be used directly within the menu)*  $\boxed{1}$  (Rect)  $\boxed{\text{G}}$  (EDITOR) and  $\boxed{1}$  (Equation). Press  $\boxed{\text{CL}}$  to exit the menu and return to the Y prompts.

The same sequence of keystrokes can be performed by touching the screen. For example, to set up the calculator in rectangular coordinates, press  $\boxed{2\text{ndF}} \boxed{\text{SET UP}}$ , and then touch **E COORD** once and double touch **1 Rect**.

Linear equations given in a form other than the slope intercept form must be manipulated to isolate the  $y$  variable. The calculator will only accept equations of the form  $y = \underline{\hspace{1cm}}$  (where  $y$  is defined explicitly in terms of  $x$ ). To enter the linear equation ( $y = 3x + 1$ ) for Y1, press  $\boxed{3} \boxed{\text{X}/\theta/\text{T}/n} \boxed{+} \boxed{1}$ . Enter the viewing window range by pressing  $\boxed{\text{ZOOM}}$ , touching **A ZOOM**, and double touching **5 Default**. This establishes the default viewing ranges for graphing equations. These ranges are  $-10 < x < 10$  and  $-10 < y < 10$ . The scale for the  $x$  and  $y$  axes are 1.



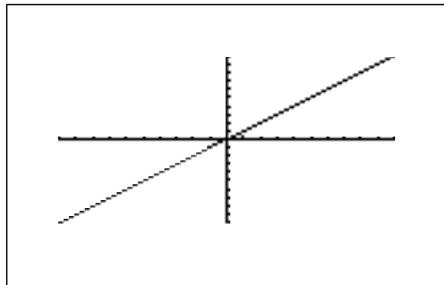
To graph another equation, press  $\boxed{\text{Y=}} \boxed{\text{CL}}$  to access and clear the Y1= prompt. Then, enter the next equation, and press  $\boxed{\text{GRAPH}}$  to view the graph. If the line does not appear in the default viewing window, press  $\boxed{\text{ZOOM}}$  and double touch **4 OUT** to enlarge the viewing window. For future use, please set the zoom factors to 5 by pressing  $\boxed{\text{ZOOM}}$ , touching **B FACTOR**, pressing  $\boxed{\text{ENTER}} \boxed{5} \boxed{\text{ENTER}} \boxed{5} \boxed{\text{ENTER}}$  and  $\boxed{\text{GRAPH}}$  to exit the zoom menu and return to the viewing screen.

## Method of Teaching

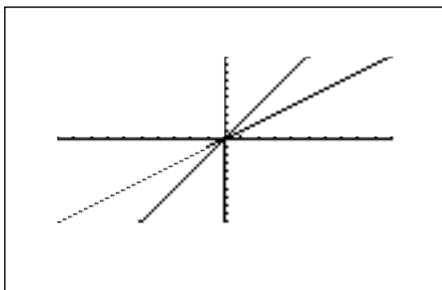
Use *Blackline Masters 1.1* and *1.2* to create overheads for graphing linear equations and investigating slope. Go over in detail how to graph linear equations, set the default viewing window, zoom out, and how to set the viewing window manually. Next, use the *Blackline Masters 1.3* and *1.4* to create worksheets for the students. Have the students graph the linear equations to explore the characteristics of the  $y$  intercept, parallel, and perpendicular. Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 1.2

Graphing a linear equation, discussed above under *Calculator Operations*, is presented on *Blackline Master 1.1*. The following study of slope appears on *Blackline Master 1.2*. Graph the equation  $y = 1x$  by pressing  $\boxed{Y=}$   $\boxed{CL}$  to remove an old  $Y1$  expression. To enter the equation press  $\boxed{X/\theta/T/n}$ . Press  $\boxed{ZOOM}$ , touch **A ZOOM**, and double touch **5 Default** to view the graph.

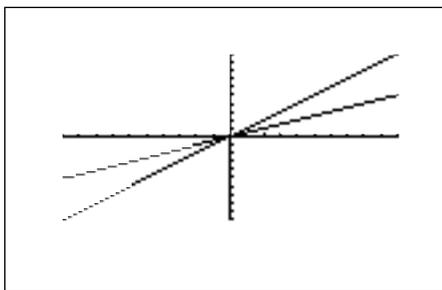


Changing the slope to 2 will result in the equation  $y = 2x$ . Enter this equation for  $Y2$  by pressing  $\boxed{Y=}$   $\boxed{\nabla}$   $\boxed{2}$   $\boxed{X/\theta/T/n}$ . To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ . The equation  $Y1=X$  is displayed first, followed by the equation  $Y2=2X$ . Notice how the "pitch" of the line has changed, with  $Y2$  becoming more steep or climbing faster. Increasing the size of the slope ( $m > 1$ ) makes the line more steep.



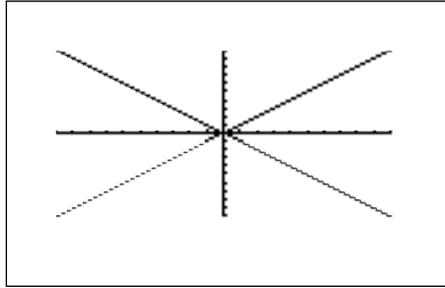
Changing the slope to  $1/2$  will result in the equation  $y = 1/2 x$ . Enter this equation for Y2 by pressing  $\boxed{Y=}$   $\blacktriangledown$   $\boxed{CL}$   $\boxed{1}$   $\boxed{a/b}$   $\boxed{2}$   $\blacktriangleright$   $\boxed{X/\theta/T/n}$ . To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .

The equation  $Y1=X$  is displayed first, followed by the equation  $Y2= 1/2 X$ . Notice how the "pitch" of the line has changed, with Y2 becoming less steep or climbing slower. Decreasing the size of the slope ( $0 < m < 1$ ) makes the line less steep.



Changing the slope to  $-1$  will result in the equation  $y = -x$ . Enter this equation for Y2 by pressing  $\boxed{Y=}$ , double touch the expression  $1/2 x$ , press  $\boxed{CL}$   $\boxed{(-)}$   $\boxed{X/\theta/T/n}$ . To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .

The equation  $Y1=X$  is displayed first, followed by the equation  $Y2= -X$ . Notice how the "pitch" of the line has changed, with Y2 decreasing (going down from left to right) instead of increasing (going up from left to right). Negative slopes ( $m < 0$ ) make the line decrease or go down from left to right.



Increasing or decreasing the descent of a line will follow the pattern set by the discussion of the positive slopes. Slopes between  $-1 < m < 0$  will lessen the descent, whereas slopes  $m < -1$  increase the descent.

### For Discussion

You and your students can discuss how to recognize that lines are the same (coincident) when expressed in different forms.

## Chapter two

# QUADRATIC EQUATIONS

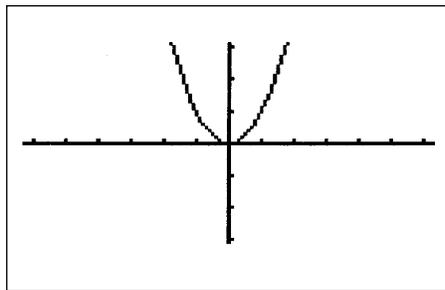
### Introducing the Topic

In this chapter, you and your students will learn how to graph quadratic equations and study their characteristics. These characteristics include the rigid translations (up/down, left/right, reflection), the vertex, and non-rigid translations (closing/opening, stretching/shrinking, or pinching/spreading). A quadratic equation of  $y$  in terms of  $x$  can be expressed by the standard form  $y = a(x - h)^2 + k$ , where  $a$  is the coefficient of the second degree term ( $y = ax^2 + bx + c$ ) and  $(h, k)$  is the vertex of the parabola formed by the quadratic equation. An equation where the largest exponent on the independent variable  $x$  is 2 is considered a quadratic equation. In graphing quadratic equations on the calculator, we will let the  $x$ -variable be represented by the horizontal axis and let  $y$  be represented by the vertical axis.

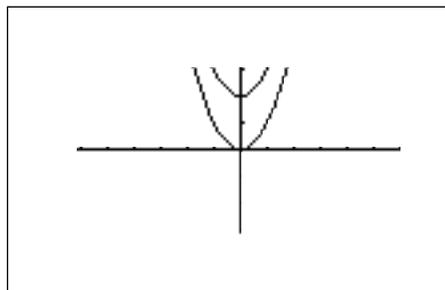
### Calculator Operations

Turn the calculator on and press  $\boxed{Y=}$ . Y prompts will appear in the viewing window. Press  $\boxed{CL}$  to remove an old Y1 expression. Press  $\boxed{ENTER}$   $\boxed{CL}$  to remove an old Y2 expression. Repeat to clear additional Y prompts. The calculator should be setup with rectangular coordinates and in equation editor mode. To complete this setup, press  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press  $\boxed{CL}$  to exit the set-up menu and return to the Y prompts.

Quadratic equations given in a form other than the standard form can be manipulated (completing the square) into standard form. This form is used for graph translations and reflections. However, the calculator will accept all quadratic equations of the form  $y = \underline{\hspace{1cm}}$  (where  $y$  is defined explicitly in terms of  $x$ ). To enter the quadratic equation ( $y = x^2$ ) for Y1, press  $\boxed{Y=}$   $\boxed{EZ}$ , touch **1**  $X^2$ , touch **A**  $Y=AX^2$ , and double touch **2**  $Y=X^2$ . Enter the viewing window range by pressing  $\boxed{ZOOM}$ , touch  $\blacktriangledown$  on the screen, and touch **7 Dec**. This establishes the decimal viewing window for graphing equations. These ranges are  $-6.3 < x < 6.3$  and  $-3.1 < y < 3.1$ . The scale for the  $x$ - and  $y$ -axes are 1.

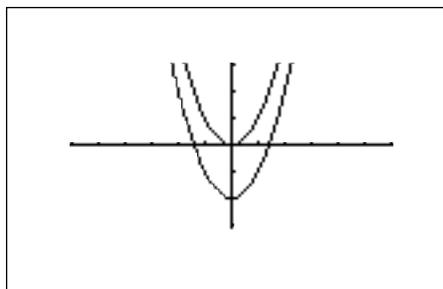


When 2 is added to the  $x^2$ , the resulting equation is  $y = x^2 + 2$ . Enter this function using the SUB feature by pressing  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{EZ}$ , touch **1**  $X^2$ , double touch **B**  $Y=AX^2 + B$ , touch **1**  $Y=AX^2 + B$ , press  $\boxed{2ndF}$   $\boxed{SUB}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{2}$   $\boxed{ENTER}$   $\boxed{GRAPH}$ . The display shows the graph of this equation and the basic  $y = x^2$ .



Notice that the addition of 2 moves the basic  $x^2$  graph up two units on the  $y$ -axis. This is equivalent to a  $k$  of 2 within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that adding a  $k > 0$  will move the basic graph up  $k$  units on the  $y$ -axis.

When  $-2$  is added to the  $x^2$ , the resulting equation is  $y = x^2 - 2$ . To change Y2 for this expression, press  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{2\text{ndF}}$   $\boxed{\text{SUB}}$   $\boxed{\text{ENTER}}$   $\boxed{(-)}$   $\boxed{2}$   $\boxed{\text{ENTER}}$   $\boxed{\text{GRAPH}}$ . The display shows the graph this equation and the basic  $y = x^2$ .



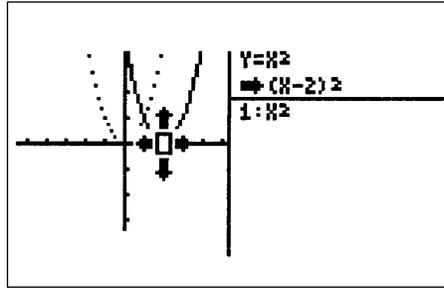
Notice that the addition of  $-2$  moves the basic  $x^2$  graph down two units on the  $y$ -axis. This is equivalent to a  $k$  of  $-2$  within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that adding a  $k < 0$  will move the basic graph down  $k$  units on the  $y$ -axis.

## Method of Teaching

Use *Blackline Masters 2.1* and *2.2* to create overheads for graphing quadratic equations and investigating translations and reflections. Go over in detail how to graph quadratic equations, use the rapid graph feature, the SUB feature, set the decimal viewing window, and use the SHIFT/CHANGE feature. Next, use *Blackline Master 2.3* to create a work sheet for the students. Have the students predict the translations and then graph the quadratic equations to verify or correct their prediction. Show the students how to trace to find the vertex of a quadratic equation's graph. Use the topics *For Discussion* to supplement the worksheets.

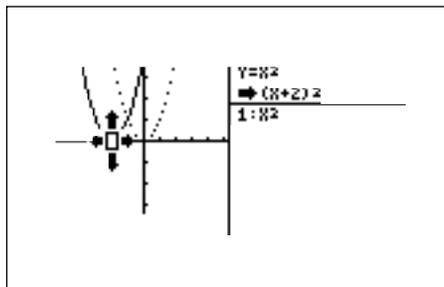
## Using Blackline Master 2.2

Vertical translations were discussed above under *Calculator Operations* and they are presented on *Blackline Master 2.1*. The following study of horizontal translations and reflections appears on *Blackline Master 2.2*. When  $-2$  is added to the  $x$  within the squaring operation ( $x^2$ ), the resulting equation is  $y = (x - 2)^2$ . To view this graph, press  $\boxed{2\text{ndF}}$   $\boxed{\text{SHIFT/CHANGE}}$ , touch **A SHIFT**, double touch **1 Y=X<sup>2</sup>**, double touch at **two on the x-axis**. The graph, the equation, and the basic  $y = x^2$  graph is shown.



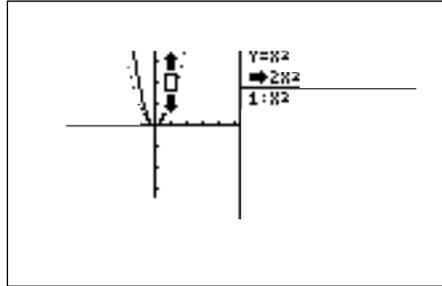
Notice that the addition of  $-2$  within the quadratic operation moves the basic  $x^2$  graph right two units on the  $x$ -axis. This is equivalent to an  $h$  of  $2$  within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that placing an  $h > 0$  in the standard form, will move the basic graph right  $h$  units on the  $x$ -axis.

When  $2$  is added to the  $x$  within the squaring operation ( $x^2$ ), the resulting equation is  $y = (x + 2)^2$ . To view this graph, press  $\boxed{2\text{ndF}} \boxed{\text{SHIFT/CHANGE}}$ , touch **A SHIFT**, double touch **1 Y=X<sup>2</sup>**, double touch at **negative two on the x-axis**. The graph, the equation, and the basic  $y = x^2$  graph is shown.



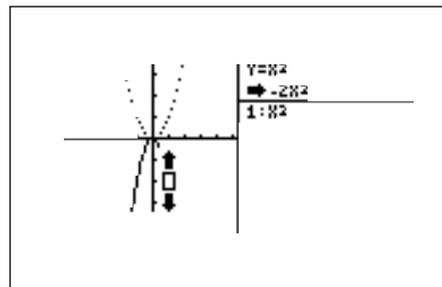
Notice that the addition of  $2$  within the quadratic operation moves the basic  $x^2$  graph left two units on the  $x$ -axis. This is equivalent to an  $h$  of  $-2$  within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that placing an  $h < 0$  within the standard form will move the basic graph left  $h$  units on the  $x$ -axis.

When  $2$  is multiplied to the  $x^2$ , the resulting equation is  $y = 2x^2$ . To view this graph, press  $\boxed{2\text{ndF}} \boxed{\text{SHIFT/CHANGE}}$ , touch **B CHANGE**, double touch **1 Y=X<sup>2</sup>**, press  $\boxed{\blacktriangle} \boxed{\text{ENTER}}$ . The graph, the equation, and the basic  $y = x^2$  graph is shown.



Notice that the multiplication of 2 pinches or closes the basic  $x^2$  graph. This is equivalent to an  $a$  of 2 within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that multiplying an  $a > 1$  will pinch or close the basic graph.

When  $-2$  is multiplied to the  $x^2$ , the resultant equation is  $y = -2x^2$ . To view this graph, press  $\boxed{2\text{ndF}} \boxed{\text{SHIFT/CHANGE}}$ , touch **B CHANGE**, double touch **1 Y=X<sup>2</sup>**, press  $\boxed{\blacktriangledown} \boxed{\blacktriangledown} \boxed{\blacktriangledown} \boxed{\text{ENTER}}$ . The graph, the equation, and the basic  $y = x^2$  graph is shown.



Notice that the multiplication of  $-2$  pinches or closes the basic  $x^2$  graph and flips it (reflects it) across the  $x$ -axis. This is equivalent to an  $a$  of  $-2$  within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that multiplying an  $a < -1$  will pinch or close the basic graph and flip it (reflect it) across the  $x$ -axis.

When 2 divides the  $x^2$ , the resulting equation is  $y = 1/2 x^2$ . Division by 2 spreads or opens the basic  $x^2$  graph. This is equivalent to an “ $a$ ” of  $1/2$  within the standard form  $y = a(x - h)^2 + k$ . This demonstrates the fact that multiplying an “ $a$ ” between 0 and 1 will spread or open the basic graph. Also, multiplying an “ $a$ ” between  $-1$  and 0 will spread or open the basic graph and flip it across the  $x$ -axis.

### For Discussion

You and your students can use the trace feature to locate the vertex of the parabola and discover that the point is  $(h, k)$ .

## Chapter three

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# FORMULAS OR LITERAL EQUATIONS

### Introducing the Topic

In this chapter, you and your students will learn how to use the graphing calculator's solver feature to evaluate formulas or literal equations. The process begins by accessing the solver feature, then entering the equation, assigning values to the known variables, and then solving for the unknown variable. Three solving methods are available within the SOLVER mode. These are the Equation method, Newton's method, and the Graphic method. The default method is the Equation method, which is used when an exact solution can be found by simple substitution.

Newton's method can be used for more complicated solutions. It implements an iterative approach to find the solution once its given a starting point. When a starting point is not readily available or multiple solutions are expected, you can use the graphic method. The graphic method plots the left and right sides of the equation and then locates the intersection(s).

One very useful feature of the EL-9600 graphing calculator is its ability to store and recall equations.

## Calculator Operations

Turn the calculator on and press **2ndF** **SOLVER** to access the solver feature. A blank screen should appear. If the screen is not blank, then press **CL** to clear the screen. Select the Equation method for solving by pressing **2ndF** **SOLVER**, and touching **A METHOD**, and double touching **1 Equation**.

Enter the formula  $P = L \left[ \frac{1 - (1 + I/12)^{-N}}{I/12} \right] - 1$

by pressing **ALPHA** **P** **ALPHA** **=** **ALPHA** **L** **(** **a/b** **1** **-** **(** **1** **+** **ALPHA** **I** **a/b** **1** **2** **▶** **)** **a<sup>b</sup>** **(-)** **ALPHA** **N** **▶** **▶** **ALPHA** **I** **÷** **1** **2** **▶** **)** **a<sup>b</sup>** **(-)** **1** **.**

$$P=L \left[ \frac{1 - \left( 1 + \frac{I}{12} \right)^{-N}}{I/12} \right] - 1$$

This equation is referred to as the amortization formula, where a loan (L) with a fixed rate of interest (I) is said to be amortized if both the loan and the interest are paid by a sequence of equal monthly payments (P). The number of equal monthly payments to be made is designated by “N”.

Press **ENTER** to view the variable list. The variables are listed in order of appearance (left to right) with payment (P) first followed by loan amount (L), interest rate (I), and number of months (N).

```
Solver:Equation
P=0
L=0
I=0
N=0
```



While viewing the variable list within the solver feature, press **CL** **CL** to return to and clear the formula entry screen. Select the Graphic method for solving by pressing **2ndF** **SOLVER**, touch **A METHOD**, and double touch **3 Graphic**. Enter the formula  $V = \pi r^2 h$  by pressing **ALPHA** **V** **ALPHA** **=** **2ndF**  **$\pi$**  **ALPHA** **R** **a<sup>b</sup>** **2** **▶** **ALPHA** **H**.

A screenshot of a calculator's formula entry screen. The screen displays the equation  $V = \pi R^2 H$  in a monospaced font. The variables V, R, and H are in uppercase, and pi is represented by the Greek letter symbol.

This equation is the formula for calculating the volume of a cylinder (V) in terms of the cylinder's height (H) and radius (R). Press **ENTER** to view the variable list. The variables are listed in order of appearance (left to right) with volume (V) first, followed by radius (R) and height (H). To find the radius of a cylinder with a volume of 30 cubic inches, and a height of 10 inches, enter the values by pressing **3** **0** **ENTER** **▼** **1** **0** **ENTER**. Press **▲** and notice the radius (R) is now highlighted by the cursor.

A screenshot of a calculator's variable list screen. The screen displays the text "Solver:Graphic" at the top. Below it, the variables and their values are listed: "V=30", "R=0", and "H=10". The variable R is highlighted with a cursor.

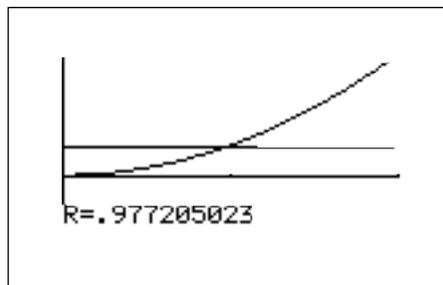
Press **2ndF** **EXE** to solve for the radius. The graphic solver will prompt you for a variable range to solve within. This range should be set to hopefully include the radius you are solving for in the problem. A quick examination of the formula shows that the radius squared will be no larger than 3 (dividing the height that we know). Therefore, one could be too small ( $1^2 = 1 < 3$ ) and two is too large ( $2^2 = 4 > 3$ ). Use the larger of the values to be safe. So, set the variable range to 0 and 2 by pressing **0** **ENTER** **2** **ENTER**.

```

Graphic solver
variable range
BEGIN=0
END=2

```

Press **2ndF** **EXE** to solve. First, the solver feature will graph the left side of the equation (volume,  $y = 30$ ). Second, the solver feature will graph the right side of the equation ( $y = 10 r^2$ ), and then will compute the intersection of the two graphs to find the solution.



Pressing **CL** will return you to the variable screen. You can now change or solve for any of the values. Save this formula by pressing **2ndF** **SOLVER**, touching **C SAVE**, pressing **ENTER**, and entering the formula name. Give the formula the name VCYL by pressing **V** **C** **Y** **L** **ENTER**.

### For Discussion

You and your students can discuss how to retrieve an equation or formula from the calculator's memory and use it to solve a problem. Also, you can discuss how to rename or delete formulas no longer desired.

### Additional Problems

1. Payments on a \$10,000 loan at 8% over 36 months.
2. Loan amount possible at 7% over 60 months with a \$300 payment.
3. Radius of a cylinder with a volume of  $200 \text{ in}^3$  and a height of 15 in.

## Chapter four

# GRAPHING POLYNOMIALS AND FINDING THE ROOTS

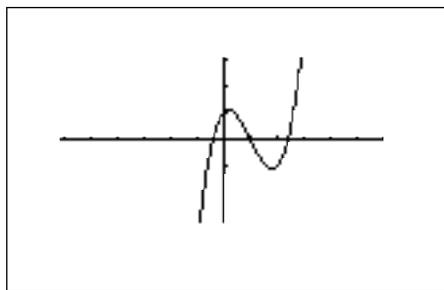
### Introducing the Topic

In this chapter, you and your students will learn how to graph polynomials and find their roots through zooming in and tracing, and through the calculate feature.

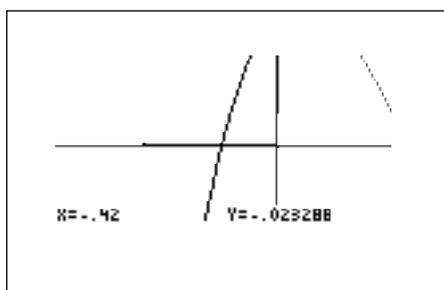
### Calculator Operations

Turn the calculator on and press  $\boxed{Y=}$ . Y prompts will appear on the viewing window. Press  $\boxed{CL}$  to remove old Y expressions. The calculator should be setup with rectangular coordinates, and in equation editor mode. To complete this setup, press  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press  $\boxed{CL}$  to exit the menu and return to the Y prompts.

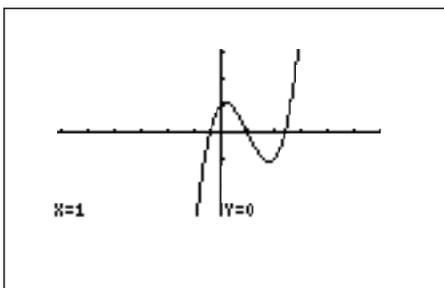
To enter the polynomial  $y = x^3 - 3x^2 + x + 1$ , press  $\boxed{EZ}$ , double touch **5 X<sup>3</sup>**, touch **A Y=AX<sup>3</sup>+BX<sup>2</sup>+CX+D**, touch **1 Y=AX<sup>3</sup>+BX<sup>2</sup>+CX+D**, press  $\boxed{2ndF}$   $\boxed{SUB}$ , and enter the coefficients by pressing  $\boxed{1}$   $\boxed{ENTER}$   $\boxed{(-)}$   $\boxed{3}$   $\boxed{ENTER}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{1}$   $\boxed{ENTER}$ . Press  $\boxed{ZOOM}$ , touch  $\blacktriangledown$  on the screen, and touch **7 Dec** to establish the decimal viewing window and view the graph.



Press **TRACE** to engage the trace feature. Press **◀** to move the cursor near the left-hand root. Make sure your zoom factors are set to 5 as suggested earlier in the book by pressing **ZOOM**, touch **B FACTOR**, press **ENTER** **5** **ENTER** **5** **ENTER**. Press **ZOOM**, touch **A ZOOM**, double touch **3 IN** to zoom in on the left-hand root. Press **TRACE** and move the tracer to approximate the root.

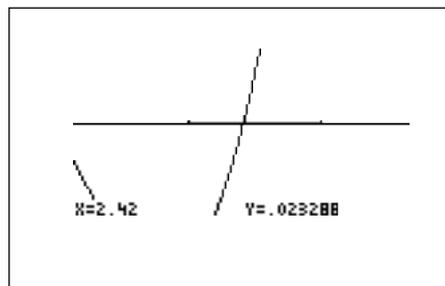


Press **ZOOM**, touch **H RCL**, double touch **2 PreWin** to return to the decimal viewing window. Press **TRACE** and move the tracer to approximate the middle root. The root is exactly  $x = 1$ , therefore no zooming is needed to find a better approximate.



Press  repeatedly to move the tracer near the right-hand root.

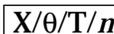
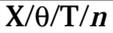
Press , touch **A ZOOM**, double touch **3 In** to zoom in. Press  and move the tracer to approximate the root.

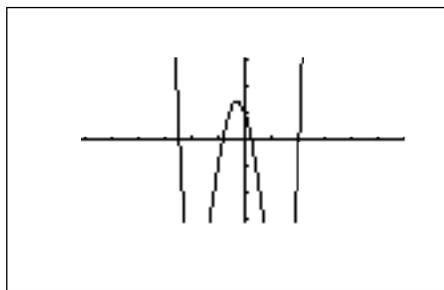


## Method of Teaching

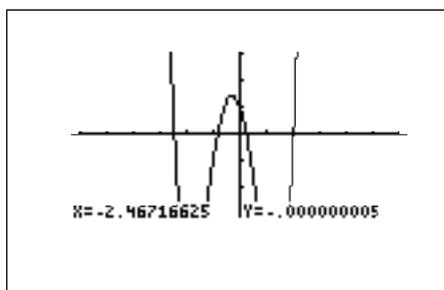
Use *Blackline Masters 4.1* and *4.2* to create overheads for graphing polynomials and finding their roots through zooming and the calculate feature. Go over in detail how to enter polynomials (both manually and rapid graph), set the viewing window, trace, set zoom factors, zoom in, and how to use the calculate feature. Next, use the *Blackline Master 4.3* to create a worksheet for the students. Have the students graph the polynomials and approximate their roots with zooming and the calculate features. Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 4.2

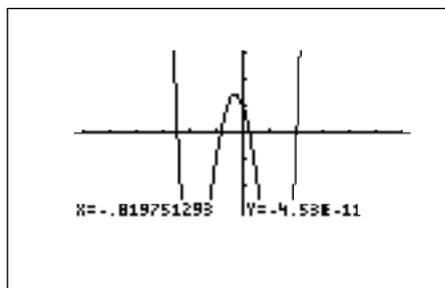
Finding roots by zooming was discussed above under *Calculator Operations* and presented on *Blackline Master 4.1*. The following discussion of the jump feature appears on *Blackline Master 4.2*. Press   to return to and clear the Y1 prompt. To enter the polynomial  $y = x^4 + x^3 - 5x^2 - 3x + 1$ , press                  . View the graph in the decimal viewing window by pressing , touching **A ZOOM**, touching  on the screen, and touching **7 Dec**.



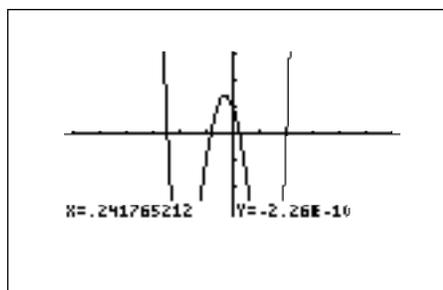
Press **TRACE** and move the cursor near the right-hand most root.  
 Press **2ndF** **CALC** to view the calculate menu. Double touch **5 X\_Incpt** and the calculator will find the left-hand root.



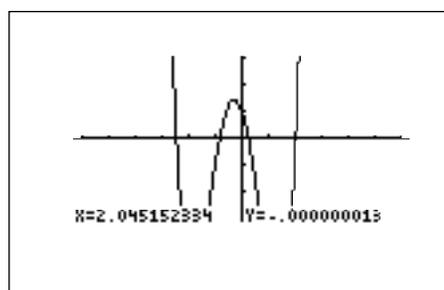
Press **2ndF** **CALC**, double touch **5 X\_Incpt** to find the next root.



Press **2ndF** **CALC**, double touch **5 X\_Incpt** to find the next root.



Press **2ndF** **CALC** , double touch **5 X\_Incpt** to find the next root.



### For Discussion

Discuss the process of zooming in more than once to achieve a better approximate for the root. Emphasize that the zooming and calculation values are normally approximates for the roots, since the roots can be irrational. Also, demonstrate the polynomial solver within tool feature. This solver will find the roots for second and third degree polynomials.

### Additional Problems

Find the roots of the following polynomials.

1.  $y = x^3 + x^2 - 2x - 1$  (zooming)
2.  $y = x^4 - 5x^2 + 3$  (zooming)
3.  $y = x^5 - 2x^4 - 4x^3 + 3x^2 + 2x - 1$  (calculate)

# Chapter five

## SOLVING A SYSTEM OF EQUATIONS

### Introduction

In this chapter, you and your students will learn how to solve a system of equations using the calculate and tool features of the EL-9600.

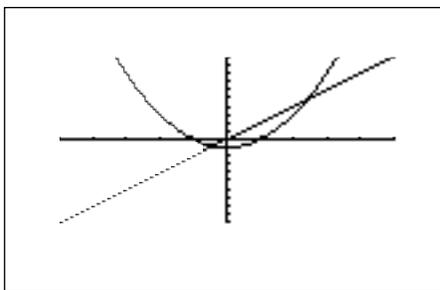
### Calculator Operations

Turn the calculator on and press  $\boxed{Y=}$ . Y prompts will appear in the viewing window. Press  $\boxed{CL}$  to clear an old Y1 expression. Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear additional Y prompts. The calculator should be setup with rectangular coordinates, and in equation editor mode. To complete this setup, press  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press  $\boxed{CL}$  to exit the menu and return to the Y prompts.

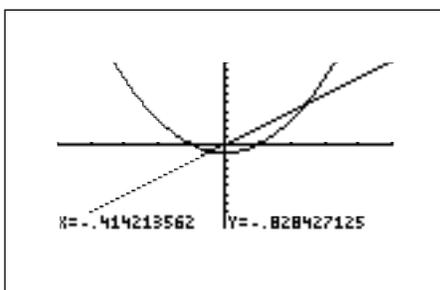
To enter the system of equations:  $y = x^2 - 1$

$$y = 2x$$

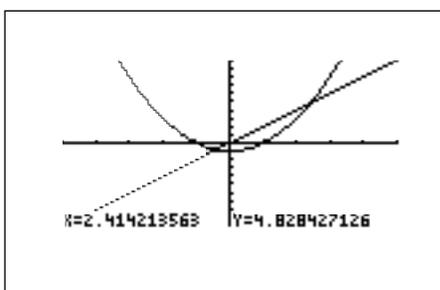
press  $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{-}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{2}$   $\boxed{X/\theta/T/n}$ . View the graphs using the rapid window by pressing  $\boxed{WINDOW}$   $\boxed{EZ}$ , touch **5**, touch  $\blacktriangleright$  on the screen, double touch **7 -5<X<5**, and double touch **4 -10<Y<10**. This approach establishes the viewing window of interest rapidly.



Press **TRACE** and move the tracer right of the right intersection.  
 Press **2ndF** **CALC** to access the calculate feature. Double touch **2 Intsct**.  
 The left-hand intersection will appear on the screen.



Press **2ndF** **CALC** to access the calculate feature again. Double touch **2 Intsct**.  
 The right-hand intersection will appear on the screen.



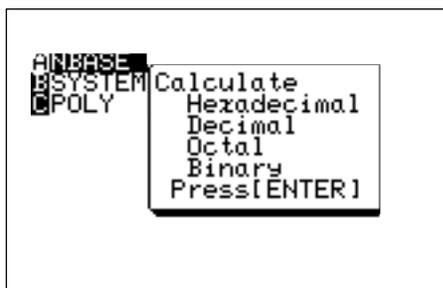
## Method of Teaching

Use *Blackline Masters 5.1* and *5.2* to create overheads for graphing systems of equations and finding their intersections through the calculate and tool features. Go over in detail how to enter equations, set the viewing window using the rapid window feature, use the calculate feature to find intersections, and how to use

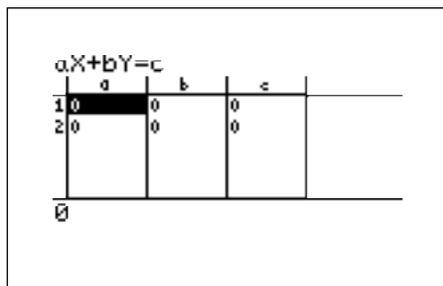
the tool feature. Next, use the *Blackline Master 5.3* to create a worksheet for the students. Have the students graph the system of equations and approximate the solutions with the calculate feature. Then, have them use the tool feature to solve a system of equations. Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 5.2

Finding solutions to a system of equations using the calculate feature was discussed above under *Calculator Operations* and presented on *Blackline Master 5.1*. The following discussion of the tool feature appears on *Blackline Master 5.2*. Press **2ndF** **TOOL** to access the tool menu.



Touch **B SYSTEM** and double touch **2 2** to view the entry screen for solving a linear-system of equations. Systems up to six variables and six equations can be solved.



To enter the system of equations:  $5x + y = 1$

$$-3x + y = -5$$

Press **5** **ENTER** **1** **ENTER** **1** **ENTER** **(-)** **3** **ENTER** **1** **ENTER** **(-)** **5** **ENTER** .

aX+bY=c			
	a	b	c
1	5	1	1
2	-3	1	-5
			-5

Press **2ndF** **EXE** to solve the system.

```

aX+bY=c
X=.75
Y=-2.75

```

### For Discussion

Discuss what to do if an intersection of the graphs does not appear on the viewing screen. Discuss what is happening if the tool feature gives you an error in its calculation (inconsistent systems/parallel lines have no solution and dependent/coincident lines have many solutions). Emphasize that solutions are normally considered approximations since some of the roots are irrational. Discuss how you know that you have found all the intersections.

### Additional Problems

Solve the following system of equations.

1.  $y = -x^2 + 2$  (calculate feature)  
 $y = x + 1$

2.  $-2x + y = -2$  (tool feature)  
 $\frac{1}{2}x - y = 1$

## Chapter six

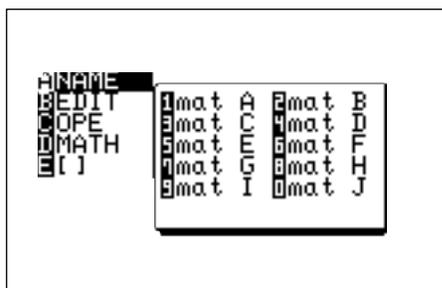
# MATRIX SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS

### Introducing the Topic

In this chapter, you and your students will learn how to solve a system of linear equations. Particularly, you will focus on a system of three linear equations, each consisting of three variables. Equations in three variables cannot be graphed on the graphing calculator. You can only find the solution of a system of equations in three variables numerically. A graphical solution is not possible.

### Calculator Operations

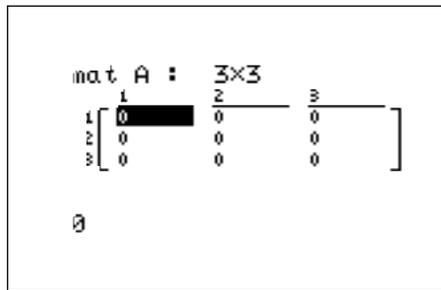
Press **MATRIX** to access a menu for entering, editing, and operating on matrices.



You can select from 10 different matrices. Touch **B EDIT** and double touch **1 mat A** to select matrix A. Now, enter the size or dimension of the matrix. We will enter the  $3 \times 3$  matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$$

Press **3** **ENTER** **3** **ENTER** to set the dimension of the matrix at three rows by three columns. The calculator will now prompt you for the matrix.



Enter the elements of the first row by pressing **1** **ENTER** **2** **ENTER** **1** **ENTER**. The calculator will move the prompt to the second row. Enter the elements of the second row by pressing **2** **ENTER** **1** **ENTER** **(-)** **1** **ENTER**. Enter the elements of the third row by pressing **1** **ENTER** **1** **ENTER** **(-)** **2** **ENTER**. The arrow keys can be used to move the prompt around the matrix. Press **2nd** **QUIT** to exit the display of matrix A.

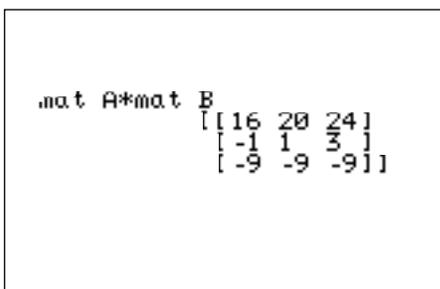
Repeat the process to enter a  $3 \times 3$  matrix B =

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Matrix multiplication can be performed if the number of columns of the first matrix is equal to the number of rows of the second matrix. In the matrix multiplication  $A \times B$ , the elements in the first row of A ( 1 2 1 ) are multiplied to the corresponding elements in the first column of B ( 1 4 7 ). The sum of these

multiplications ( $1 \cdot 1 + 2 \cdot 4 + 1 \cdot 7$ ) is placed in the 1,1 (first row, first column) position of the resulting matrix. This process is repeated until each row of A has been multiplied to each column of B.

To multiply the matrices A and B together, press **MATRIX**, touch **A NAME**, double touch **1 mat A**, press **×** **MATRIX**, double touch **2 mat B**, and press **ENTER**. The resulting matrix is shown below.



## Method of Teaching

Use *Blackline Masters 6.1* and *6.2* to create overheads for entering matrices, multiplying matrices, finding inverse matrices, and solving systems of equations. Go over in detail the derivation of the matrix solution (presented below), and identity matrices. Next, use *Blackline Master 6.3* to create a worksheet for the students. Have the students solve systems of linear equations using matrices. Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 6.2

The identity matrix I is defined to be a square matrix ( $n \times n$ ) where each position on the diagonal is 1 and all others are 0. For example, the  $3 \times 3$  identity matrix is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

The calculator will directly establish an identity matrix of a given size by pressing **MATRIX**, touching **C OPE**, double touching **05 identity**, and pressing **3 ENTER**. To save the identity matrix in matrix C, press **STO** **MATRIX**, touch **A NAME**, touch **3 mat C**, and press **ENTER**.

```

identity 3
[[1 0 0]
 [0 1 0]
 [0 0 1]]

Ans→mat C
[[1 0 0]
 [0 1 0]
 [0 0 1]]

```

Confirm that the identity matrix is stored in matrix C by pressing **MATRIX** , touching **B EDIT**, and double touching **3 mat C**.

```

mat C : 3x3
1 1 2 3
2 [ 1 0 0 ]
3 [ 0 1 0 ]
  [ 0 0 1 ]

```

Press **2ndF** **QUIT** to exit the matrix editor and press **CL** to clear the screen.

An inverse matrix  $A^{-1}$  is a matrix that when multiplied to A results in the identity matrix I ( $A^{-1} \times A = I$ ). The graphing calculator can find  $A^{-1}$  quickly. Derivation of  $A^{-1}$  by hand is not discussed here. Find the inverse of the square matrix A by pressing **MATRIX** , touching **A NAME**, double touching **1 mat A**, pressing **2ndF**  **$x^{-1}$**  **ENTER** . The resulting matrix is shown below. Press **▶** to see more of the matrix.

```

mat A-1
[[-.1666666666 .8333333333...
 [ .5 -1.5
 [ .1666666666 .1666666666...

```

```

mat A-1
...6666 .83333333333333 -.5]
...6666 -1.5 .5]
...6666 .16666666666666 -.5]]

```

Some square matrices have no inverse and will generate error statements when calculating the inverse.

To solve the system of equations  $x + 2y + z = 8$   
 using matrices:  $2x + y - z = 1$   
 $x + y - 2z = -3$

Use the matrix A entered previously as the coefficient matrix, and enter the constants on the right side of the equal sign into matrix B ( $3 \times 1$ ), where B =

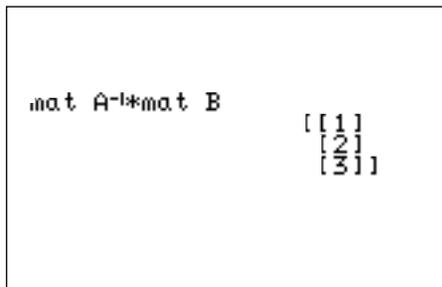
$$\begin{bmatrix} 8 \\ 1 \\ -3 \end{bmatrix}$$

Press **2ndF** **QUIT** to exit the display of the B matrix. The solution matrix X is found by multiplying  $A^{-1} B$ . This multiplication is derived from the equation:

$$\begin{aligned} AX &= B \\ A^{-1} A X &= A^{-1} B \text{ (multiply both sides by } A^{-1} \text{)} \\ I X &= A^{-1} B \text{ (} A^{-1}A = I, \text{ identity matrix)} \\ X &= A^{-1} B \text{ (} I X = X \text{)} \end{aligned}$$

Note, the multiplication is "order sensitive" and you will not get the correct answer by multiplying  $B A^{-1}$ .

Multiply  $A^{-1} B$  by pressing **MATRIX**, touching **A NAME**, double touching **1 mat A**, pressing **2ndF**  **$x^{-1}$**   **$\times$**  **MATRIX**, double touching **2 mat B**, and pressing **ENTER**. The solution matrix will appear:



The 1 is the X coordinate, 2 the Y coordinate, and 3 the Z coordinate of the solution point.

### For Discussion

You and your students can discuss why the graphing calculator cannot graph equations of three variables, the derivation of the  $X = A^{-1}B$  formula for solving a system of linear equations, the size of the matrix generated by multiplication of two matrices, and multiplication by the identity matrix. Introduce the students to the TOOL feature and its system solver. *Blackline Master 6.3* shows the method for solving systems of equations using the TOOL feature.

### Additional Problems

Solve the following systems of equations:

1.  $x + 2y + 3z = 4$   
 $2x - y - 2z = 0$   
 $x - 3y - 3z = -2$

2.  $x - y + z = 3$   
 $2x - y + z = 4$   
 $x + 2y - z = -1$

## Chapter seven

# INEQUALITIES AND SYSTEMS OF INEQUALITIES

### Introducing the Topic

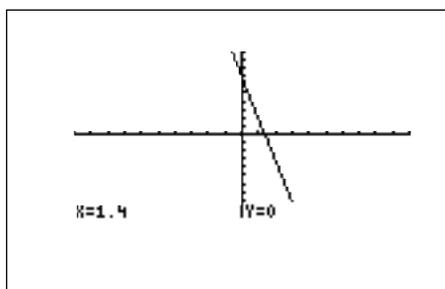
In this chapter you and your students will explore graphical techniques to solve inequalities and systems of inequalities. Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often very difficult to solve algebraically.

### Calculator Operations

To solve an inequality means to find all values that make the inequality true. Two methods of finding these values for one-variable inequalities are illustrated in this chapter.

The first method involves rewriting the inequality so that the right-hand side of the inequality is 0 and the left-hand side is a function of  $x$ . Then, find the solution to  $f(x) < 0$  by determining where the graph of  $f(x)$  is below the  $x$ -axis, or find the solution to  $f(x) > 0$  by determining where the graph of  $f(x)$  is above the  $x$ -axis, or find the solution to  $f(x) \leq 0$  by determining where the graph of  $f(x)$  is below or on the  $x$ -axis, or find the solution to  $f(x) \geq 0$  by determining where the graph of  $f(x)$  is above or on the  $x$ -axis.

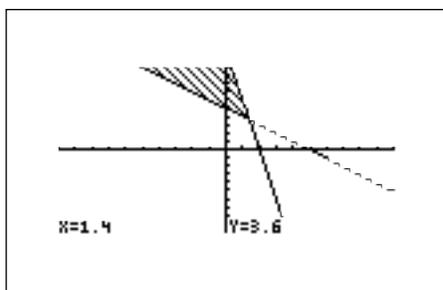
For example, to solve  $3(4 - 2x) \geq 5 - x$ , rewrite it as  $3(4 - 2x) - 5 + x \geq 0$  and determine the values of  $x$  where the function  $y = 3(4 - 2x) - 5 + x$  is on or above the  $x$ -axis. To do this, press  $\boxed{Y=}$   $\boxed{CL}$  and enter  $3(4 - 2X) - 5 + X$  in the Y1 location. Set the viewing window of the graph by pressing  $\boxed{ZOOM}$ , touch **A ZOOM**, and double touch **5 Default**. You should be able to clearly view the  $x$ -intercept. Locate the  $x$ -intercept at the point  $(1.4, 0)$  with the keystrokes  $\boxed{TRACE}$   $\boxed{2ndF}$   $\boxed{CALC}$ , and double touch **5 X\_Incpt**.



Since the graph is above the  $x$ -axis to the left of the  $x$ -intercept, the solution to the inequality  $3(4 - 2x) - 5 + x \geq 0$  is all values of  $x$  such that  $x \leq 1.4$ .

The second method involves graphing each side of the inequality as an individual function. Then, find the solution to  $f(x) < g(x)$  by determining where the graph of  $f(x)$  is below the graph of  $g(x)$ , or find the solution to  $f(x) > g(x)$  by determining where the graph of  $f(x)$  is above the graph of  $g(x)$ , or find the solution to  $f(x) \leq g(x)$  by determining where the graph of  $f(x)$  is below or intersects the graph of  $g(x)$ , or find the solution to  $f(x) \geq g(x)$  by determining where the graph of  $f(x)$  is above or intersects the graph of  $g(x)$ .

For example, to solve the inequality  $3(4 - 2x) \geq 5 - x$ , press  $\boxed{Y=}$   $\boxed{CL}$ , enter  $3(4 - 2X)$  for Y1 and  $5 - X$  for Y2. Set the viewing window by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, and double touching **5 Default**. Next, shade the set of points that make the inequality true by pressing  $\boxed{2ndF}$   $\boxed{DRAW}$ , touching **G SHADE**, and double touching **1 Set** to access the Set Shade screen. Since the inequality you are solving is  $Y1 \geq Y2$  the solution is where the graph of Y1 is "on the top" and Y2 is "on the bottom." Do this by touching **Y2** on the screen, pressing  $\blacktriangleright$ , and touching **Y1** on the screen. Press  $\boxed{GRAPH}$  to view the shaded region. Press  $\boxed{TRACE}$   $\boxed{2ndF}$   $\boxed{CALC}$ , and double touch **2 Intsct** to find where the graphs intersect.



Since the shaded region is to the left of  $x = 1.4$ , the solution to the inequality  $3(4 - 2x) \geq 5 - x$  is all values of  $x$  such that  $x \leq 1.4$ .

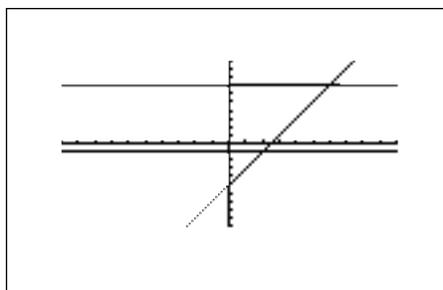
### Double Inequalities

The solution to a system of two inequalities in one variable consists of all values of the variable that make each inequality in the system true.

A system such as  $\begin{cases} 2x - 5 \geq -1 \\ 2x - 5 \leq 7 \end{cases}$ , where the same expression appears on both

inequalities, is commonly referred to as a “double” inequality and is often written in the form  $-1 \leq 2x - 5 \leq 7$ . Be certain that students notice the order of the inequality signs and that the double inequality is only used to indicate an expression in  $x$  “trapped” in between two values. Unless both inequality signs are pointing in the same direction, the double inequality expression is meaningless and should not be used. Also, by the transitive property of inequalities,  $a$  must be less than or equal to  $b$  in the inequality  $a \leq f(x) \leq b$  or  $b \geq f(x) \geq a$ .

Turn off the shading by pressing  $\boxed{2\text{ndF}} \boxed{\text{DRAW}}$ , touching **G SHADE**, and double touching **2 INITIAL**. To clear any previously entered functions press  $\boxed{Y=}$   $\boxed{\text{CL}}$ . Enter  $Y1 = -1$ ,  $Y2 = 2X - 5$ , and  $Y3 = 7$ . Press  $\boxed{\text{ZOOM}}$ , touch **A ZOOM**, and double touch **5 Default** to view the line  $y = 2x - 5$  between the lines  $y = -1$  and  $y = 7$ .



Press **TRACE** **2ndF** **CALC** , double touch **2 Intsct** to find the point of intersection of the lines  $y = 2x - 5$  and  $y = -1$  at  $(2, -1)$ . Press **▲** to move the tracer to the  $y = 7$  line. Press **2ndF** **CALC** and double touch **2 Intsct** to find  $y = 2x - 5$  and  $y = 7$  at  $(6, 7)$ . The solution to the “double” inequality  $-1 \leq 2x - 5 \leq 7$  consists of all values of  $x$  in between, and including, 2 and 6 (i.e.,  $x \geq 2$  and  $x \leq 6$ ). The solution is  $2 \leq x \leq 6$ .

### Systems of Two-Variable Inequalities

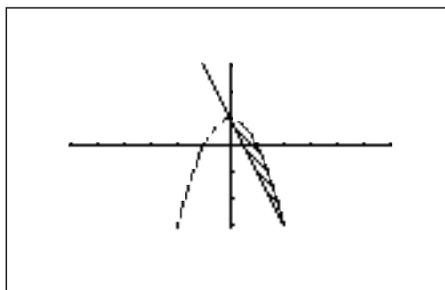
The solution region of a system of two-variable inequalities consists of all points  $(a, b)$  such that when  $x = a$  and  $y = b$ , all inequalities in the system are true.

For example, to graph the solution region for the system  $\begin{cases} 2x + y \geq 1 \\ x^2 + y \leq 1 \end{cases}$ , first

rewrite each inequality in the system so that the left-hand-side is  $y$ :

$$\begin{cases} y \geq 1 - 2x \\ y \leq 1 - x^2 \end{cases}$$

Press **Y=** **CL** and enter  $1 - 2X$  in  $Y1$  and  $1 - X^2$  in  $Y2$ . Next, shade the points that have a  $y$ -value less than  $Y2$  and greater than  $Y1$ . Do this by pressing **2ndF** **DRAW** , touching **G SHADE**, double touching **1 Set**, touch **Y1** on the screen, press **►** , and touch **Y2** on the screen. Graph the system by pressing **ZOOM** , touching **A ZOOM**, touching **▼** on the screen, and touching **7 Dec**.



### Method of Teaching

Use *Blackline Master 7.1* to create an overhead or handout for using the first method to graphically solve an inequality. Remind students of the proper algebraic procedure to write the inequality in the form  $f(x) < 0$ . Next, use *Blackline Master 7.2* to create an overhead or handout for using the second

method to graphically solve an inequality. Demonstrate the process of entering the left- and right-hand sides of the inequality in the Y1 and Y2 locations and the process for shading the solution.

When using *Blackline Master 7.3*, discuss the meaning of a “double” inequality and its definition in terms of a system of two single-variable inequalities. Use *Blackline Master 7.4* to create an overhead or handout for solving a system of two-variable inequalities. Emphasize that while the solution to a single-variable inequality is one dimensional, the solution to a system of inequalities in two variables is a region of points.

Use *Blackline Master 7.5* to create a worksheet for the students. Have the students choose either method to solve the inequalities in Problem 3. Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 7.5

The first method for solving an inequality is presented on *Blackline Master 7.1*. The second method for solving an inequality is presented on *Blackline Master 7.2*. The method for solving a double inequality is presented on *Blackline Master 7.3*. The method for solving a system of two-variable inequalities is presented on *Blackline Master 7.4*. Use *Blackline Master 7.5* to create a worksheet for the students to practice what they have learned with *Blackline Masters 7.1-4*.

### For Discussion

Discuss the use of the “free-moving” cursor to graphically determine if a specific point is in the solution region of a system of two-variable inequalities. After viewing the shaded solution region for the system of inequalities, determine if a point  $(x, y)$  is a solution by pressing the arrow keys and moving the cursor to the approximate location of the point and see if it is within the shaded region. Viewing the region using the decimal window or multiples of those settings allows for exact free-moving cursor location of most points. Students should also algebraically confirm that the point  $(x, y)$  satisfies both inequalities by substituting those values in the original inequalities.

## Additional Problems

1. Solve  $6(x - 2) - 2(x - 5) < 3x$  using the first method.
2. Solve  $\frac{4x + 3}{8} - 2 \geq \frac{x + 3}{4}$  using the second method.
3. For which values of  $x$  are the following inequalities true?
  - a.  $x^3 > 6 + 7x$
  - b.  $2(x - 1)^2 - 3x > -3(1 + x)$
  - c.  $10 + x > 4 - \frac{x}{5}$
  - d.  $3 < 5 - 2x < 6$

4. Solve the system 
$$\begin{cases} \frac{(5x)}{8} + 2 \leq 1 \\ \frac{(5x)}{8} + 2 \geq \frac{1}{2} \end{cases}$$

Express your answer as a double inequality.

5. a. Graph the solution region for the system of inequalities 
$$\begin{cases} 2x + 5y > 6 \\ 3x + 2.5y < 1.5 \end{cases}$$
  - b. Are the points  $(.6, .5)$  and/or  $(3.5, 0)$  solutions to this system?
6. a. Graph the solution region for the system of inequalities 
$$\begin{cases} x^2 + y < 4 \\ y > (x - 1)(x + 2) \end{cases}$$
  - b. Are the points  $(-2, 0)$  and/or  $(-1, 2.5)$  solutions to this system?

# Chapter eight

## ABSOLUTE VALUE FUNCTIONS, EQUATIONS, AND INEQUALITIES

### Introducing the Topic

This chapter provides a brief introduction to the absolute value function and discusses methods to graphically solve equations and inequalities involving the absolute value function.

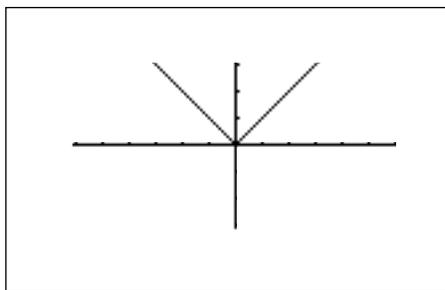
### Calculator Operations

The absolute value of a real number  $x$  is defined by the following:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0. \end{cases}$$

The symbol for absolute value is found in the MATH menu. When using the EQUATION editor, the absolute value appears just as we write it on paper,  $| \quad |$ .

Press  $\boxed{Y=}$   $\boxed{CL}$  to clear the Y1 prompt. Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear the remaining prompts. Enter the function  $f(x) = |x|$  in Y1 by pressing  $\boxed{MATH}$ , touching **B NUM**, double touching **1 abs(**, and pressing  $\boxed{X/\theta/T/n}$ . View the graph by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and touching **7 Dec**.

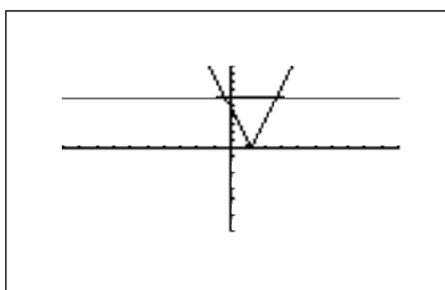


Notice, by the definition of this function with reinforcement from the graphical view, that the domain of  $f(x) = |x|$  is the set of all real numbers and the range is the set of non-negative real numbers.

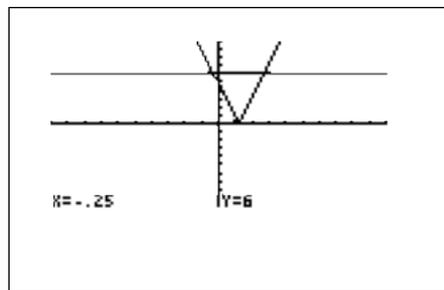
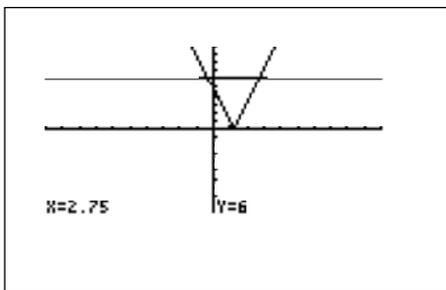
### Absolute Value Equations

If  $n$  is a positive number, there are two solutions to the equation  $|f(x)| = n$  because there are exactly two numbers with the absolute value equal to  $n$ :  $n$  and  $-n$ . The existence of two distinct solutions is clear when the equation is solved graphically.

For example, to solve  $|5 - 4x| = 6$ , press  $\boxed{Y=}$   $\boxed{CL}$  to clear the Y1 prompt. Enter  $|5 - 4x|$  in Y1 with the keystrokes  $\boxed{MATH}$ , touch **B NUM**, double touch **1 abs(**, and press  $\boxed{5}$   $\boxed{-}$   $\boxed{4}$   $\boxed{X/\theta/T/n}$ . Next, press  $\boxed{ENTER}$   $\boxed{CL}$   $\boxed{6}$  to enter 6 in Y2. Press  $\boxed{ZOOM}$ , touch **A ZOOM**, double touch **5 Default** to view the two points of intersection of the absolute value graph and the horizontal line  $y = 6$ .



Press  $\boxed{TRACE}$   $\boxed{2ndF}$   $\boxed{CALC}$  and double touch **2 Intsct** to find one point of intersection of the two graphs. Press  $\boxed{2ndF}$   $\boxed{CALC}$  and double touch **2 Intsct** again to find the other point of intersection.



Therefore, the solution to the equation  $|5 - 4x| = 6$  consists of the two values  $-0.25$  and  $2.75$ . Note that although it is not as intuitively obvious, the solution could also be obtained by finding the  $x$ -intercepts of the function  $y = |5x - 4| - 6$ .

### Absolute Value Inequalities

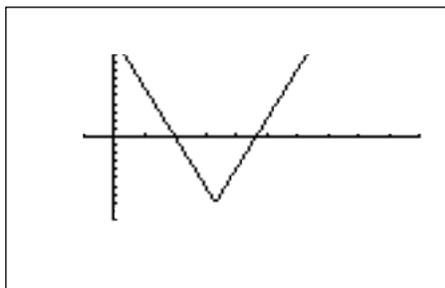
Absolute value inequalities are of the form  $|f(x)| < k$ ,  $|f(x)| \leq k$ ,  $|f(x)| > k$ , or  $|f(x)| \geq k$ . The graphical solution to an absolute value inequality is found using either method discussed in Chapter 7.

To solve  $\left|20 - \frac{(6x)}{5}\right| < 8$  using the first method, rewrite the inequality so that the right-hand side of the inequality is zero:  $\left|20 - \frac{(6x)}{5}\right| - 8 < 0$ .

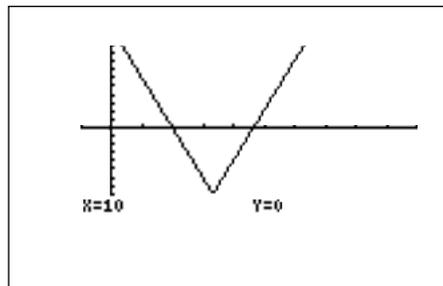
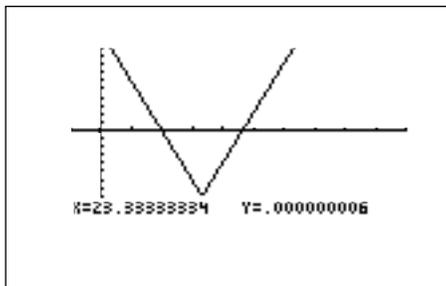
Press  $\boxed{Y=}$  and clear all the  $Y$  prompts. Enter the left-hand side of the inequality  $\left|20 - \frac{(6x)}{5}\right| - 8$  in  $Y1$  by pressing  $\boxed{\text{MATH}}$ , touching **NUM**, double touching  $\boxed{1}$  **abs**(, pressing  $\boxed{2}$   $\boxed{0}$   $\boxed{-}$   $\boxed{a/b}$   $\boxed{6}$   $\boxed{X/\theta/T/n}$   $\boxed{\blacktriangleright}$   $\boxed{5}$   $\boxed{\blacktriangleright}$   $\boxed{\blacktriangleright}$   $\boxed{-}$   $\boxed{8}$ .

Choose a viewing window by pressing  $\boxed{\text{WINDOW}}$   $\boxed{\text{EZ}}$ , double touching **6** on the screen, double touching **4**  $-5 < X < 50$ , and double touching **4**  $-10 < Y < 10$ .

The  $x$ -intercepts of the graph are clearly visible.



Press **TRACE** and move right of the right intercept. Press **2ndF** **CALC** and double touch **5 X\_Incpt** to find the first  $x$ -intercept. Press **2ndF** **CALC** and touch **5 X\_Incpt** to find the second  $x$ -intercept.

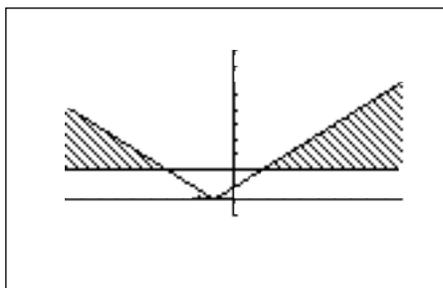


Since the graph is below the  $x$ -axis for  $x$  in between the two  $x$ -intercepts, the solution is  $10 < x < 23.333$ .

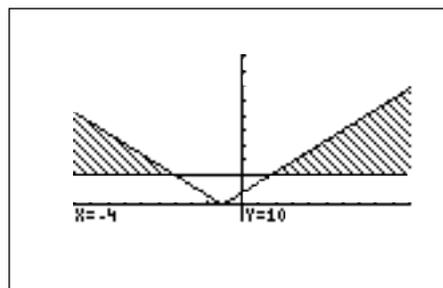
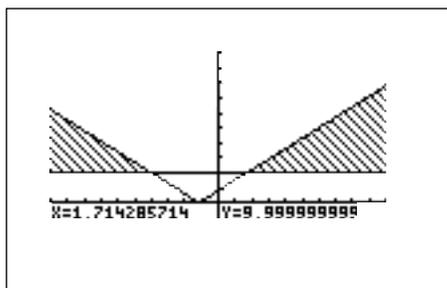
To use the second method to solve the inequality  $|3.5x + 4| > 10$ , first press **Y=** **CL** to clear the Y1 prompt. Press **ENTER** **CL** to clear additional prompts. Enter the function  $|3.5x + 4|$  in Y1 by pressing **MATH**, touching **B NUM**, double touching **1 abs(**, and pressing **3** **•** **5** **X/θ/T/n** **+** **4** **ENTER**. Enter 10 in Y2 by pressing **1** **0**.

Access the Set Shade screen by pressing **2ndF** **DRAW**, touching **G SHADE**, and double touching **1 SET**. Since Y2 is the function "on the bottom," touch **Y2** on the screen, and since Y1 is the function "on the top," press **▶** and touch **Y1** on the screen.

Choose a viewing window by pressing **WINDOW** **EZ**, double touching **2** on the screen, double touching **5 -10 < X < 10**, and double touching **3 -5 < Y < 50**.



Press **TRACE** **2ndF** **CALC**, double touch **2 Intsct** to locate a point of intersection. Repeat to find the other. The intersections are  $(-4, 10)$  and  $(1.714, 10)$ . The solution of the inequality  $|3.5x + 4| \geq 10$  is all values of  $x$  such that  $x \leq -4$  or  $x \geq 1.714$ .



## Method of Teaching

Use *Blackline Masters 8.1* and *8.2* to create overheads or handouts for investigating the absolute value function. Next, use *Blackline Masters 8.3, 8.4* and *8.5* to create overheads or handouts for graphically solving absolute value equations and inequalities.

*Blackline Master 8.6* should be used to create a worksheet for students. Students should choose either method for solving the absolute value inequalities. Pay careful attention to the written form of the answer.

Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 8.1

Use *Blackline Master 8.1* to demonstrate how to evaluate an absolute value on the home or computation screen. The students will then graph  $y = |x|$ , followed by graphing  $y = |x - 1|$  and  $y = |x| - 1$ . Discuss with them the effect on the graph of  $y = |x|$  when 1 is subtracted from the  $x$ -term ( $y = |x - 1|$ ) and when 1 is subtracted from the function ( $y = |x| - 1$ ).

*Blackline Masters 8.3, 8.4* and *8.5* are discussed above under Calculator Operation.

## For Discussion

You and your students can use the calculator, with its SHIFT/CHANGE and SUB features, to explore for specific values of A, B, and C in the function

$$y = A|x + B| + C:$$

1. the effect of the value of A on the graph of the absolute value function,
2. the effect of the value of B on the graph of the absolute value function, and
3. the effect of the value of C on the graph of the absolute value function.

Include positive, negative, integer, and fractional values of A, B, and C in your discussion, and have the students look for patterns in the graphs as one constant is changed and the others remain fixed.

## Additional Problems

1. Is  $|-2+7| = |-2| + |7|$ ? Evaluate each side of the equation to check your answer.

Is  $|x+y| = |x| + |y|$  for all real numbers  $x$  and  $y$ ? If not, when will

$$|x+y| = |x| + |y|?$$

2. Is  $\left|\frac{6-9}{1+3}\right| = \frac{|6-9|}{|1+3|}$  Evaluate each side of the equation to check your answer.

Investigate with more examples, and decide if you think  $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$  for all real numbers  $x$  and all real numbers  $y \neq 0$ .

3. Solve  $\left|3 - \frac{(2x)}{5}\right| = 4$ .

4. Find the values of  $x$  for which  $|4x+5| < 3$ .

5. Solve  $\left|\frac{(2-5x)}{3}\right| \geq 2$ .

# Chapter nine

## RATIONAL FUNCTIONS

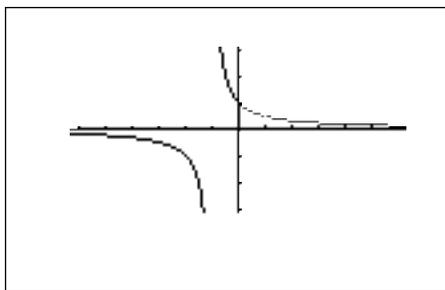
### Introducing the Topic

This chapter provides a brief introduction to rational functions and discusses methods to graphically solve equations and inequalities involving rational functions.

### Calculator Operations

A rational function  $f(x)$  is defined as the quotient  $\frac{p(x)}{q(x)}$  where  $p(x)$  and  $q(x)$  are two polynomial functions such that  $q(x) \neq 0$ . The domain of any rational function consists of all values of  $x$  such that the denominator  $q(x)$ , is not zero. A rational function consists of branches separated by vertical asymptotes, and the values of  $x$  that make the denominator  $q(x) = 0$  but do not make the numerator  $p(x) = 0$  are where the vertical asymptotes occur.

Consider the rational function  $f(x) = \frac{(x-1)}{(x^2-1)}$ . Press  $\boxed{Y=}$   $\boxed{CL}$  to clear the Y1 prompt. Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear additional Y prompts. Enter  $Y1 = \frac{(x-1)}{(x^2-1)}$  by pressing  $\boxed{a/b}$   $\boxed{X/\theta/T/n}$   $\boxed{-}$   $\boxed{1}$   $\boxed{\blacktriangleright}$   $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{-}$   $\boxed{1}$ . Graph in the decimal window by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.



Notice that since  $f(x)$  can be written as  $\frac{(x-1)}{(x+1)(x-1)}$ , the domain consists of all real numbers  $x$  such that  $x \neq 1$  and  $x \neq -1$ . The function consists of two branches separated by the vertical asymptote (the line  $x = -1$ ). However, there is no vertical asymptote where  $x = 1$  since this value of  $x$  also makes the numerator zero. Look closely at the graph and observe the "hole" in it at  $x = 1$ . Press **TRACE** to place the tracer in the center of the screen. Press **▶** repeatedly to move the tracer to the coordinates  $X = 0.9$ ,  $Y = 0.5263$ . Press **▶** to see that the calculator does not display a value for  $Y$  at  $X = 1$  since 1 is not in the domain of this rational function. Press **▶** once more and observe that the displayed coordinates are  $X = 1.1$ ,  $Y = 0.48$ . Note that the hole will be seen only when the window is a "friendly" or decimal window. This means you can trace to the exact  $x$ -value. (For instance, if you graphed this function in a window of  $-5 \leq x \leq 5$ ,  $x = 1$  is not one of the possible trace values for  $x$  and the hole will not be seen.)

The  $x$ -intercepts of a rational function  $f(x)$ , if there are any, occur at the  $x$ -values that make the numerator  $p(x)$ , but not the denominator  $q(x)$ , zero. The  $y$ -intercept occurs at  $f(0)$ . Notice that there are no  $x$ -intercepts for the graph of  $f(x) = \frac{(x-1)}{(x^2-1)}$ . The  $y$ -intercept can easily be found at  $x = 0$ ,  $y = 1$  by pressing **2ndF** **CALC** **( $x^2-1$ )** and double touching **6 Y\_Incpt**.

Rational functions also have horizontal asymptotes, lines of the form  $y = k$  ( $k$  a constant) such that the function gets arbitrarily close to, but does not cross, the horizontal asymptote when  $|x|$  is large. (Some rational functions composed of higher-degree polynomials might cross horizontal asymptotes for small values of  $x$ ). Rational functions may also have an oblique asymptote, a line that is neither horizontal nor vertical, but which the function gets closer and closer to, but does not touch, as  $|x|$  gets very large. Algebraic methods for finding the equations of horizontal and oblique asymptotes are covered in most algebra texts and are not given in this chapter.

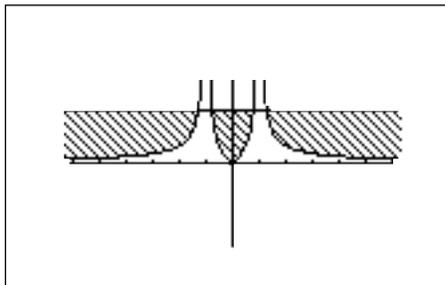
Students can observe from the graph of  $f(x) = \frac{(x-1)}{(x^2-1)}$  that the line  $y = 0$  is very likely a horizontal asymptote of  $f(x)$ .

### Rational Function Inequalities

Inequalities involving rational functions are traditionally solved algebraically using sign charts. However, the graphical process of solving rational function inequalities is the same as solving other inequalities. Simply use either method discussed in Chapters 7 and 8.

For instance, to solve the inequality  $\left| \frac{x}{1-x^2} \right| \leq 2$  using the second method (intersection method with shading), press  $\boxed{Y=}$   $\boxed{CL}$  to clear the Y1 prompt. Enter the left-hand side of the inequality in Y1 by pressing  $\boxed{MATH}$ , touching **B NUM**, double touching **1 abs(**, pressing  $\boxed{a/b}$   $\boxed{X/\theta/T/n}$   $\boxed{\blacktriangleright}$   $\boxed{1}$   $\boxed{-}$   $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{ENTER}$ . Enter 2 in Y2 by pressing  $\boxed{2}$   $\boxed{ENTER}$ .

Press  $\boxed{2ndF}$   $\boxed{DRAW}$  touch **G SHADE**, double touch **1 SET** to access the Set Shade screen. Since Y1 is the value “on the bottom” (the smaller of the two), touch Y1 on the screen first, press  $\boxed{\blacktriangleright}$ , and since Y2 is the function “on the top” (the larger of the two), touch Y2 on the screen second. View the graph by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.



Press  $\boxed{2ndF}$   $\boxed{CALC}$  and double touch **2 Intsct** to locate the  $x$ -value of the leftmost point of intersection,  $x = -1.281$ . Press  $\boxed{2ndF}$   $\boxed{CALC}$  and touch **2 Intsct**, three more times to locate the  $x$ -values of the other three points of intersection of Y1 and Y2:  $x = -0.781$ ,  $0.781$ , and  $1.281$ . The solution is all values of  $x$  such that  $x \leq -1.281$  or  $-0.781 \leq x \leq 0.781$  or  $x \geq 1.281$ . Turn off the shading by pressing  $\boxed{2ndF}$   $\boxed{DRAW}$ , touching **G SHADE**, and double touching **2 INITIAL**.

## Method of Teaching

Use *Blackline Masters 9.1 and 9.2* to create overheads or handouts for investigating rational functions and the solution to rational inequalities. Explain horizontal and vertical asymptotes and how to find the equation of a vertical asymptote.

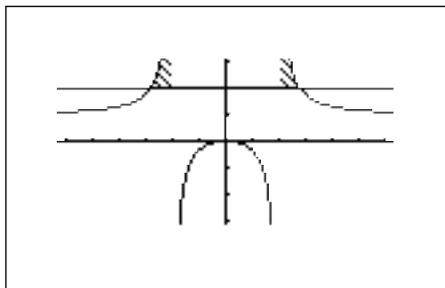
The number of branches students should expect to see on the graph of a rational function is one more than the number of vertical asymptotes. Students should therefore always determine the number of vertical asymptotes to know when they are viewing a complete graph of a rational function.

Next, use *Blackline Master 9.3* to create overheads or handouts for graphing rational functions and graphically solving inequalities involving rational functions. Use the topics *For Discussion* to supplement the worksheets.

## Using Blackline Master 9.3

Graphing rational functions and inequalities was discussed under *Calculator Operations* and presented on *Blackline Masters 9.1 and 9.2*.

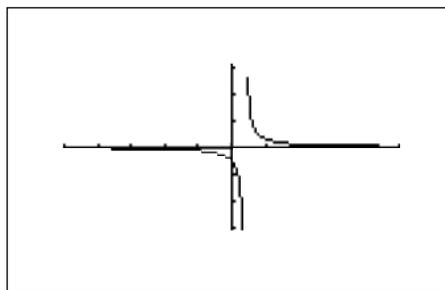
In Activity 1, the instructions for entering the left- and right-hand sides of the rational inequality are given. After entering the rational expression in Y1 and 2 in Y2, have the students shade. Since Y2 is "on the bottom," touch Y2 on the screen first, press , and since Y1 is the function "on the top," touch Y1 on the screen second. Before the graph is drawn, have the students discuss what they expect to see: an  $x$ -intercept at  $x = 0$  and two vertical asymptotes at  $x = -2$  and  $x = 2$  giving three branches for the rational function. Graph in the decimal window, changing  $y$ -max to 6.2 to provide a good view of the graph.



In Activity 2, have students discuss in groups what they should expect to see for the graph of the rational function  $y = \frac{(x-3)}{(x^2-5x+6)}$  before drawing the graph. If necessary, remind students that when a common factor appears in both the numerator and denominator of a rational function, that factor results in a hole in the graph, not a vertical asymptote.

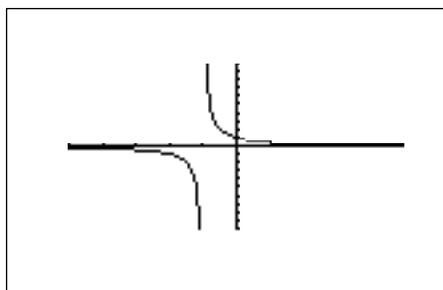
Since  $\frac{(x-3)}{(x^2-5x+6)} = \frac{(x-3)}{(x-2)(x-3)}$ , the graph should have two branches since there is only one vertical asymptote, the line  $x = 2$ . There is a hole in the graph at  $x = 3$ .

Also have students verify by pressing **TRACE** **2ndF** **CALC** and double touching **5 X\_Incpt**, that there are no  $x$ -intercepts for this rational function. Have them view the graph in the decimal viewing window and then zoom out horizontally by setting Xmin to -25 and Xmax to 25 with Xscl= 5 to estimate from the graphs that the line  $y = 0$  is very likely a horizontal asymptote.



### For Discussion

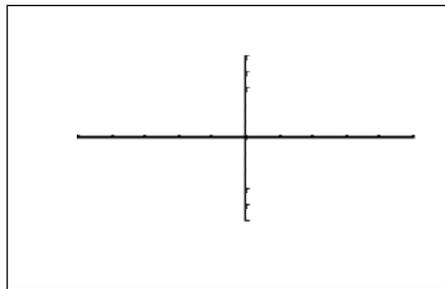
Use the rapid window and trace features to explore the graph of a rational function near a vertical asymptote. For instance, after completing *Blackline Master 9.1*, have the students redraw the graph by pressing **WINDOW** **EZ**, touching **5**, touching  $\blacktriangledown$  on the screen, double touching **7**  $-5 < X < 5$ , and double touching **4**  $-10 < Y < 10$ .



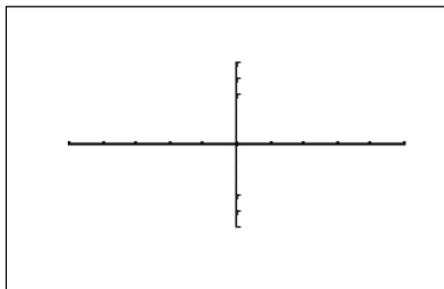
Next, have them press **TRACE** and press **◀** repeatedly to move the tracer along the graph. Observe the  $y$ -values as the graph begins to scroll vertically, going up to the point  $(-0.95, 21)$  and then jumping down to the other branch at  $(-1.03, -31.5)$ .

### Additional Problems

1. a. Graph the rational function  $f(x) = \frac{(x-3)}{x}$ .



- b. What is the domain of  $f(x)$ ?  
 c. Give the equation(s) of the vertical asymptote(s) of  $f(x)$ .  
 d. Find the  $x$ - and  $y$ -intercepts of  $f(x)$ .  
 e. Estimate, using your graph, the horizontal asymptote of  $f(x)$ .
2. a. Graph the rational function  $g(x) = \frac{x(x+1)}{(x^2-1)(x+2)}$ .



- b. What is the domain of  $g(x)$ ?  
 c. Give the equation(s) of the vertical asymptote(s) of  $g(x)$ , and draw each vertical asymptote with a dashed line on the graph above.  
 d. Find the  $x$ - and  $y$ -intercepts of  $g(x)$ .  
 e. Estimate, using your graph, the horizontal asymptote of  $g(x)$ . Draw on the graph above, using a dashed line, the horizontal asymptote.

3. Solve the inequality  $\frac{1}{(x+1.8)^2} \leq 1.6$ .

4. Determine the value of  $x$  for which  $\frac{(x^2+x-2)}{2(x-1)} > 0$ .

# Chapter ten

## CONIC SECTIONS

### Introducing the Topic

When a right circular cone is cut by a plane, the intersection is a curve called a conic section. This chapter illustrates how to graph conic sections called parabolas, circles, ellipses, and hyperbolas. The parametric mode of the calculator is discussed in the procedure for graphing parabolas.

### Calculator Operations

#### Parabolas

Quadratic functions of the form  $y = ax^2 + bx + c$  and their graphs were discussed in Chapter 2. The graphs of quadratic equations are called parabolas.

Sometimes the quadratic equations take on the form of  $x = ay^2 + by + c$ .

Consider, for example, drawing the graph of  $x = y^2 - 2$ . There is a problem entering this equation in the calculator graphing list for two reasons: a) it is not a function, and only functions can be entered in the Y= list locations, and b) the functions entered in the Y= list must be in terms of  $x$ , not  $y$ . There are, however, two methods you can use to draw the graph of a parabola whose equation does not represent a function:

Method 1: Consider the “top” and “bottom” halves of the parabola as two different parts of the graph because each individually is a function. Solve the equation of the parabola for  $y$  and enter the two parts (that individually are functions) in two locations of the  $Y=$  list.

Method 2: Choose the parametric graphing mode of the calculator and enter the parametric equations of the parabola.

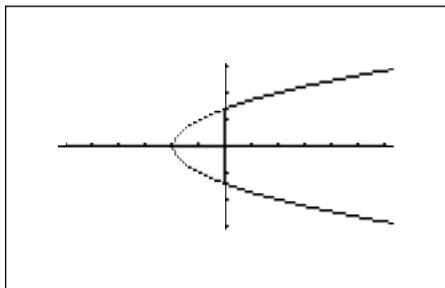
Let’s use both methods to graph the parabola  $x = y^2 - 2$ . To use Method 1, first solve the equation for  $y$ :

$$\begin{aligned}x &= y^2 - 2 \\x + 2 &= y^2 \\y &= \pm \sqrt{(x+2)}\end{aligned}$$

The graph of the equation  $y = \sqrt{(x+2)}$  is the “top half” of the parabola and the graph of the equation  $y = -\sqrt{(x+2)}$  gives the “bottom half.”

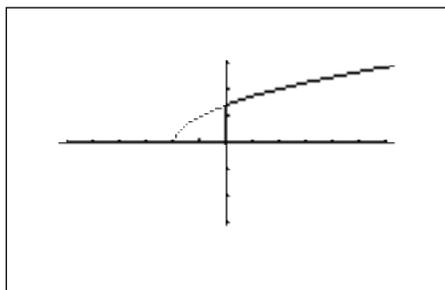
Press  $\boxed{Y=}$   $\boxed{CL}$  to clear the  $Y1$  prompt. Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear additional prompts. Enter  $\sqrt{(x+2)}$  in  $Y1$  with the keystrokes  $\boxed{2ndF}$   $\boxed{\sqrt{\quad}}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{2}$ .

Press  $\boxed{ENTER}$  and enter  $-Y1 = -\sqrt{(x+2)}$  in  $Y2$  with the keystrokes  $\boxed{(-)}$   $\boxed{VARS}$ , touch **A EQVARS**, press  $\boxed{ENTER}$ , and double touch **1 Y1**. View the graph by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.



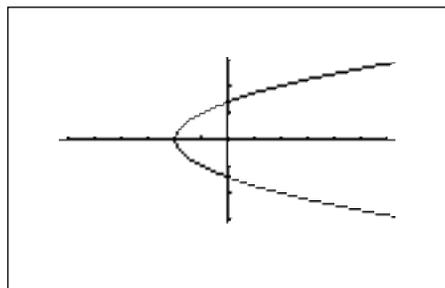
When using Method 2 to draw the graph, it is not necessary to algebraically solve the equation for  $y$ . Change to parametric mode by pressing  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, and double touch **2 Param**. Press  $\boxed{2ndF}$   $\boxed{QUIT}$  to exit the set-up screen. Press  $\boxed{Y=}$   $\boxed{CL}$  to clear the  $X1T$  prompt.

Parametric representations are equation pairs ( $x = F(t)$ ,  $y = F(t)$ ) that have X and Y each expressed in terms of a third parameter, T. To rewrite  $x = y^2 - 2$  in parametric form, simply let  $y = T$  and substitute in  $x = y^2 - 2$  to obtain  $x = T^2 - 2$ . Enter  $X1T = T^2 - 2$  in the calculator by pressing  $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{-}$   $\boxed{2}$   $\boxed{ENTER}$ . (Notice that in parametric mode, pressing  $\boxed{X/\theta/T/n}$  produces T, not X.) Enter  $Y1T = T$  by pressing  $\boxed{X/\theta/T/n}$ . View the graph by pressing  $\boxed{ZOOM}$ , touch **A ZOOM**, touch  $\blacktriangleright$  on the screen, and double touch **7 Dec**.



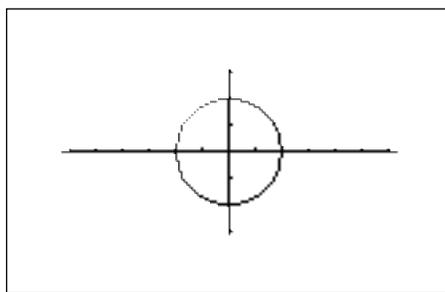
Notice that only half of the parabola is drawn. To understand why this occurs, press  $\boxed{TRACE}$  and  $\blacktriangleright$  to move the tracer right. The tracer starts at  $T=0$  and increases. Substitute  $T=0$  in  $X1T$  and  $Y1T$  to obtain  $X1T = -2$  and  $Y1T = 0$ . This is the first point graphed by the calculator.

To see the rest of the parabola, press  $\boxed{WINDOW}$  and adjust the Tmin. Set Tmin to  $-6$  by pressing  $\boxed{(-)}$   $\boxed{6}$   $\boxed{ENTER}$ . The Tstp determines how much T changes before the next point is plotted on the graph. Even though the dimensions of the viewing screen are still controlled by Xmin, Xmax, Ymin, and Ymax, the T settings determine the number of points that are plotted. Press  $\boxed{GRAPH}$  to view the complete parabola.



## Circles

The standard form of a circle of radius  $r$ , centered at the origin, is  $x^2 + y^2 = r^2$ . To graph the circle  $x^2 + y^2 = 4$  using Method 1, first return to the rectangular graphing mode by pressing  $\boxed{2\text{ndF}} \boxed{\text{SET UP}}$ , touching **E COORD**, double touching **1 Rect**, and pressing  $\boxed{2\text{ndF}} \boxed{\text{QUIT}}$ . Press  $\boxed{Y=}$   $\boxed{\text{CL}}$  to clear the Y1 prompt. Enter  $Y1 = \sqrt{4 - X^2}$  (the top half) by pressing  $\boxed{2\text{ndF}} \boxed{\sqrt{\phantom{x}}} \boxed{4} \boxed{-}$   $\boxed{\text{X}/\theta/\text{T}/n}$   $\boxed{x^2}$   $\boxed{\text{ENTER}}$  and then enter  $Y2 = -\sqrt{4 - X^2}$  (the bottom half) by pressing  $\boxed{\text{CL}}$   $\boxed{-}$   $\boxed{\text{VARS}}$ , touch **A EQVARS**, press  $\boxed{\text{ENTER}}$ , touch **A XY**, and double touch **1 Y1**. View the graph by pressing  $\boxed{\text{ZOOM}}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.



The standard equation of a circle of radius  $r$  that is centered at a point  $(h, k)$  is  $(x - h)^2 + (y - k)^2 = r^2$ . In order to put an equation in standard form so that you can use entry method 1, it is necessary to solve the equation for  $y$ . You therefore need to use the process of completing the square. Notice that it is only necessary to complete the square on the  $y$ -term in order to enter the equation in the  $Y=$  list. For instance, in order to draw the graph of the circle  $x^2 - 2x + y^2 + 4y = 2$ , proceed as follows:

$$x^2 - 2x + y^2 + 4y = 2$$

$$x^2 - 2x + y^2 + 4y + 4 = 2 + 4$$

$$x^2 - 2x + (y+2)^2 = 6$$

$$(y+2)^2 = 6 - x^2 + 2x$$

$$y + 2 = \pm \sqrt{6 - x^2 + 2x}$$

$$y = \pm \sqrt{6 - x^2 + 2x} - 2$$

Place all variable terms on the left and the constant term on the right-hand side of the equation.

Complete the square on the  $y$ -term.

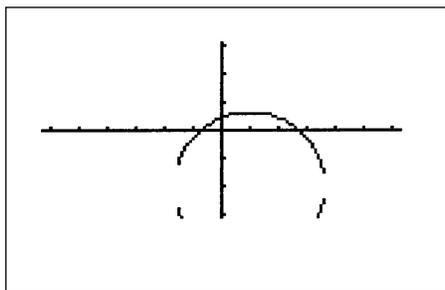
Express the terms in  $y$  as a perfect square.

Leave only the term involving  $y$  on the left hand side.

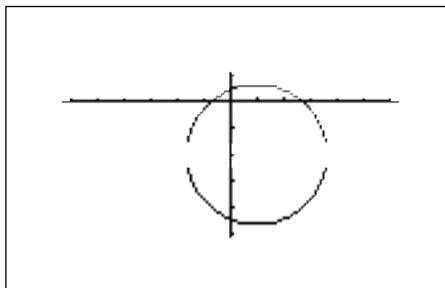
Take the square root of both sides.

Solve for  $y$ .

Notice that if you enter  $Y1 = \sqrt{(6 - x^2 + 2x)} - 2$  and  $Y2 = -Y1$ , you will not get the graph of a circle because the “±” does not go with the “-2”. To avoid a lot of keystrokes, enter  $Y1 = \sqrt{(6 - x^2 + 2x)}$ ,  $Y2 = Y1 - 2$  and  $Y3 = -Y1 - 2$ . You now have the top portion and the bottom portion of the circle in  $Y2$  and  $Y3$ . Since these are the only two functions you want to graph, “turn off”  $Y1$  so that it will not graph by pressing  $\blacktriangle$  to move the blinking cursor to the  $Y1$  expression. Press  $\blacktriangleleft$  to position the blinking cursor over the = sign, and press  $\boxed{\text{ENTER}}$  so that “=” is no longer darkened. Press  $\boxed{\text{ZOOM}}$ , touch  $\blacktriangledown$  on the screen, and double touch **7 Dec**.



This is not a very good graph. We must adjust the screen to see the bottom part of the circle. Do this using the rapid zoom feature by pressing  $\boxed{\text{EZ}}$ , double touching  $\blacktriangledown$  on the bottom of the screen twice, and double touching  $\blacktriangledown$  on the top of the screen twice. Press  $\boxed{\text{GRAPH}}$  to view the graph in the new window.



The circle looks like a circle because it is displayed in the decimal window. Adjusting this window equivalently will keep the window "friendly" and a circle will appear as a circle. Also, zooming in or out from the decimal window will also keep the window "friendly." (The small separation on the sides is unavoidable in some views.)

Method 2, parametric graphing, is not discussed in this chapter for conics other than parabolas. Parametric graphing of circles, ellipses, and hyperbolas is possible, but the method requires a knowledge of trigonometry.

### Ellipses

The standard equation for an ellipse whose center is at the point  $(h, k)$  with major and minor axes of length  $a$  and  $b$  is  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ .

Method 1 for graphing using the function mode is exactly the same as the circle: you must solve for  $y$ , completing the square if necessary, and enter the “top” half and the “bottom” half in two locations of the Y= list.

### Hyperbolas

The standard equation for a hyperbola can take one of two forms:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad \text{with vertices at } (h \pm a, k) \text{ or}$$
$$\frac{(x-k)^2}{b^2} - \frac{(y-h)^2}{a^2} = 1 \quad \text{with vertices at } (h, k \pm b).$$

Method 1 for graphing using the function mode is exactly the same as the circle and the ellipse: you must solve for  $y$  and enter the “top” half and the “bottom” half in two locations of the Y= list. Solving for  $y$  will involve completing the square if the equation is not in the standard form.

## Method of Teaching

Use *Blackline Master 10.1* to create an overhead or handout for investigating graphing of parabolas using function graphing and parametric graphing. Next, use *Blackline Master 10.2* to create an overhead or handout describing the procedure to graph a circle. *Blackline Master 10.3* should be used to create a worksheet for students. Use the topics *For Discussion* to supplement the worksheet.

## Using Blackline Master 10.3

For problem 3 on *Blackline Master 10.3*, students may prefer not to complete the square and graph this parabola in parametric mode by letting  $Y1T=T$  and  $X1T = 4 - 2T+T^2$ . For problem 4 on *Blackline Master 10.3*, students should complete the square on the  $y$ -term to obtain  $y^2 - 6y + 9 = x^2 + 2x + 3 + 9$  and then  $(y-3)^2 = x^2 + 2x + 12$ . Then graph the hyperbola using rectangular coordinates with

$Y1 = (x^2 + 2x + 12)$ ,  $Y2 = Y1 + 3$ , and  $Y3 = -Y1 + 3$ . (They should turn  $Y1$  off before graphing.) Zoom out several times from a decimal window to view the graph.

### For Discussion

Review the procedure for completing the square on the  $y$ -term so that the equation representing a circle, ellipse, or hyperbola can be solved for  $y$  and entered in the  $Y=$  list.

### Additional Problems

1. Use rectangular mode to graph  $x - y^2 = 4$ .
2. Use parametric mode to graph  $y^2 + x + 1 = 0$ .

Classify each of the following as a parabola, circle, ellipse, or hyperbola and construct the graph.

3.  $3x^2 + y = 9$
4.  $2x + (y + 1)^2 = 6$
5.  $x^2 + y^2 - 6y = 8$
6.  $\frac{y^2}{4} - \frac{(x + 1)^2}{16} = 1$

# CONTENTS OF REPRODUCIBLE BLACKLINE MASTERS

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Use these reproducible Blackline Masters to create handouts, overhead transparencies, and activity worksheets.

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# LINEAR EQUATIONS

## Steps for graphing a linear equation

1. Turn the calculator on and press  $\boxed{Y=}$ .
2. Press  $\boxed{CL}$  to remove an old Y1 expression.
3. Set the calculator to rectangular coordinates and in equation editor mode, by pressing  $\boxed{2ndF}$   $\boxed{SET UP}$   $\boxed{E}$  (COORD)  $\boxed{1}$  (Rect)  $\boxed{G}$  (EDITOR) and  $\boxed{1}$  (Equation). Press  $\boxed{CL}$  to exit the menu and return to the Y prompts.
4. Enter the linear equation ( $y = 3x + 1$ ) for Y1 by pressing  $\boxed{3}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{1}$ .
5. Enter the viewing window range by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, and double touching **5 Default**. This establishes the default viewing ranges for graphing equations. These ranges are  $-10 < x < 10$  and  $-10 < y < 10$ . The scale for the  $x$  and  $y$  axes are 1.
6. To graph another equation, press  $\boxed{Y=}$   $\boxed{CL}$  to access and clear the Y1= prompt.
7. Enter the next equation, and press  $\boxed{GRAPH}$  to view the graph.
8. If the line does not appear in the default viewing window, press  $\boxed{ZOOM}$  and double touch **4 OUT** to enlarge the viewing window.
9. For future use, please set the zoom factors to 5 by pressing  $\boxed{ZOOM}$ , touching **B FACTOR**, pressing  $\boxed{ENTER}$   $\boxed{5}$   $\boxed{ENTER}$   $\boxed{5}$   $\boxed{ENTER}$  and  $\boxed{GRAPH}$  to exit the zoom menu and return to the viewing screen.

1.2

# LINEAR EQUATIONS

## Characteristics of Slope

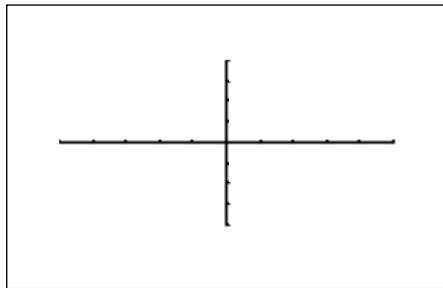
- Graph the equation  $y = 1x$  by pressing  $\boxed{Y=}$   $\boxed{CL}$  to remove an old Y1 expression, and press  $\boxed{X/\theta/T/n}$  to enter the equation.
- Press  $\boxed{ZOOM}$ , touch **A ZOOM**, and double touch **5 Default** to view the graph.
- Changing the slope to 2 will result in the equation  $y = 2x$ .  
Enter this equation for Y2 by pressing  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{2}$   $\boxed{X/\theta/T/n}$ .
- To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .  
How has the line been changed?
- Changing the slope to  $\frac{1}{2}$  will result in the equation  $y = \frac{1}{2}x$ . Enter this equation for Y2 by pressing  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{CL}$   $\boxed{1}$   $\boxed{a/b}$   $\boxed{2}$   $\boxed{\blacktriangleright}$   $\boxed{X/\theta/T/n}$ .
- To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .  
How has the line been changed?  
\_\_\_\_\_
- Changing the slope to  $-1$  will result in the equation  $y = -x$ .  
Enter this equation for Y2 by pressing  $\boxed{Y=}$ , double touch the expression  $\frac{1}{2}x$ , press  $\boxed{CL}$   $\boxed{(-)}$   $\boxed{X/\theta/T/n}$ .
- To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .  
How has the line been changed?  
\_\_\_\_\_

1.3

# LINEAR EQUATIONS

## Activity 1 Characteristics of the $y$ -intercept

- Graph the equation  $y = x$  by pressing  $\boxed{Y=}$   $\boxed{CL}$  to remove an old Y1 expression, and press  $\boxed{X/\theta/T/n}$  to enter the equation.
- Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear Y2 or additional prompts.
- Use the default viewing range, and press  $\boxed{ZOOM}$ , touch **A ZOOM**, double touch **5 Default** to view the graph.
- Adding 2 will result in the equation  $y = x + 2$ . Enter this equation for Y2 by pressing  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{2}$ .
- To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .  
How has the line been changed?  
\_\_\_\_\_
- Subtracting 2 will result in the equation  $y = x - 2$ . Enter this equation for Y2 by pressing  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{CL}$   $\boxed{X/\theta/T/n}$   $\boxed{-}$   $\boxed{2}$ .
- To view both the graphs on the same coordinate axes, press  $\boxed{GRAPH}$ .  
How has the line been changed?  
\_\_\_\_\_
- Sketch the three linear equations on the coordinate axes provided below:



1.4

# LINEAR EQUATIONS

## Activity 2 Parallel and Perpendicular Lines

- Enter the equations  $Y1 = 3X + 1$  and  $Y2 = 3X + 2$  by pressing  $\boxed{Y=}$   $\boxed{CL}$   $\boxed{3}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{CL}$   $\boxed{3}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{2}$  .
- Use the default viewing range, and press  $\boxed{ZOOM}$  , touch **A ZOOM**, and double touch **5 Default** to view the graphs. What do you notice about the two lines?  
\_\_\_\_\_
- These line are called parallel since they have an equal slope but different  $y$ -intercepts. These lines will not intersect.
- Enter the equations  $Y1 = 3X - 1$  and  $Y2 = -1/3 X + 1$  by pressing  $\boxed{Y=}$   $\boxed{CL}$   $\boxed{3}$   $\boxed{X/\theta/T/n}$   $\boxed{-}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{CL}$   $\boxed{(-)}$   $\boxed{1}$   $\boxed{a/b}$   $\boxed{3}$   $\boxed{\blacktriangleright}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{1}$  .
- Press  $\boxed{GRAPH}$  to view the graphs. What do you notice about the two lines?  
\_\_\_\_\_
- These line are called perpendicular since they have slopes that are negative reciprocals of each other ( $m_1 = -1/m_2$ ). Notice, these intersecting lines form four equal angles.
- Graph two lines with unequal slopes (not negative reciprocals). What do you see? Are the lines parallel, perpendicular or neither? If they intersect, what angles are formed by the intersection?  
\_\_\_\_\_

## 2.1

**QUADRATIC EQUATIONS****Graphing and translations of quadratic equations**

1. Turn the calculator on and press  $\boxed{Y=}$ . Press  $\boxed{CL}$  to remove an old Y1 expression. Press  $\boxed{ENTER}$   $\boxed{CL}$  to remove an old Y2 expression.
2. The calculator should be setup with rectangular coordinates, and in equation editor mode. To complete this setup, press  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press  $\boxed{CL}$  to exit the set-up menu and return to the Y prompts.
3. To enter the quadratic equation ( $y = x^2$ ) for Y1, press  $\boxed{Y=}$   $\boxed{EZ}$ , touch **1 X<sup>2</sup>**, touch **A Y=AX<sup>2</sup>**, and double touch **2 Y=X<sup>2</sup>**. Enter the viewing window range by pressing  $\boxed{ZOOM}$ , touch  $\blacktriangledown$  on the screen, and touch **7 Dec**.
4. When 2 is added to  $x^2$ , the resulting equation is  $y = x^2 + 2$ . Enter this function using the SUB feature by pressing  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{EZ}$ , touch **1 X<sup>2</sup>**, double touch **B Y=AX<sup>2</sup> + B**, touch **1 Y=AX<sup>2</sup> + B**, press  $\boxed{2ndF}$   $\boxed{SUB}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{2}$   $\boxed{ENTER}$   $\boxed{GRAPH}$ . What does the addition of 2 do?
5. When -2 is added to  $x^2$ , the resulting equation is  $y = x^2 - 2$ . To change Y2 for this expression, press  $\boxed{Y=}$   $\boxed{\blacktriangledown}$   $\boxed{2ndF}$   $\boxed{SUB}$   $\boxed{ENTER}$   $\boxed{(-)}$   $\boxed{2}$   $\boxed{ENTER}$   $\boxed{GRAPH}$ . What does the addition of -2 do?
6. Summarize the effect of  $k$  within the standard equation  $y = a(x - h)^2 + k$ .

## 2.2

**QUADRATIC EQUATIONS****Graphing and translations of quadratic equations**

1. When  $-2$  is added to  $x$  within the squaring operation ( $x^2$ ), the resulting equation is  $y = (x - 2)^2$ . To view this graph, press **2ndF** **SHIFT/CHANGE**, touch **A SHIFT**, double touch **1 Y=X<sup>2</sup>**, double touch at two on the  $x$ -axis. What does the addition of  $-2$  within the quadratic operation do to the basic  $x^2$  graph?
2. When  $2$  is added to  $x$  within the squaring operation ( $x^2$ ), the resulting equation is  $y = (x + 2)^2$ . To view this graph, press **2ndF** **SHIFT/CHANGE**, touch **A SHIFT**, double touch **1 Y=X<sup>2</sup>**, double touch at **negative two on the  $x$ -axis**. What does the addition of  $2$  within the quadratic operation do to the basic  $x^2$  graph?
3. When  $2$  is multiplied to  $x^2$ , the resulting equation is  $y = 2x^2$ . To view this graph, press **2ndF** **SHIFT/CHANGE**, touch **B CHANGE**, double touch **1 Y=X<sup>2</sup>**, press **▲** **ENTER**. What does the multiplication of  $2$  do to the basic  $x^2$  graph?
4. When  $-2$  is multiplied to the  $x^2$ , the resulting equation is  $y = -2x^2$ . To view this graph, press **2ndF** **SHIFT/CHANGE**, touch **B CHANGE**, double touch **1 Y=X<sup>2</sup>**, press **▼** **▼** **▼** **ENTER**. What does the multiplication of  $-2$  do to the basic  $x^2$  graph?
6. Extend the above by proposing what happens when  $2$  or  $-2$  divides the  $x^2$ , and the resultant equation is  $y = \frac{1}{2}x^2$  or  $y = -\frac{1}{2}x^2$ .

## 2.3

**QUADRATIC EQUATIONS****Translations and reflections of quadratic equations**

In each of the following quadratic equations given in standard form, predict the vertical and horizontal translations, opening and closing of the parabolas, and possible reflections. Then verify or correct your prediction using the EL-9600 graphing calculator.

1.  $y = x^2 + 1$

2.  $y = \frac{1}{3}x^2$

3.  $y = -x^2$

4.  $y = (x - 3)^2$

5.  $y = 2(x + 1)^2$

6.  $y = -\frac{1}{4}x^2 - 3$

7.  $y = 3(x - 1)^2 + 2$

8.  $y = -\frac{1}{5}(x + 2)^2 - 1$

## 3.1

# FORMULAS OR LITERAL EQUATIONS

## Steps for solving an equation using the equation method

1. Turn the calculator on and press  $\boxed{2\text{ndF}}$   $\boxed{\text{SOLVER}}$  to access the solver feature. A blank screen should appear. If the screen is not blank, then press  $\boxed{\text{CL}}$  to clear the screen.
2. Select the Equation method for solving by pressing  $\boxed{2\text{ndF}}$   $\boxed{\text{SOLVER}}$ , and touching **A METHOD**, and double touching 1 Equation.
3. Enter the formula  $P = L \left[ \frac{1 - (1 + I/12)^{-N}}{I/12} \right] - 1$

Press  $\boxed{\text{ALPHA}}$   $\boxed{\text{P}}$   $\boxed{\text{ALPHA}}$   $\boxed{=}$   $\boxed{\text{ALPHA}}$   $\boxed{\text{L}}$   $\boxed{(}$   $\boxed{\text{a/b}}$   $\boxed{1}$   $\boxed{-}$   $\boxed{(}$   $\boxed{1}$   
 $\boxed{+}$   $\boxed{\text{ALPHA}}$   $\boxed{\text{I}}$   $\boxed{\text{a/b}}$   $\boxed{1}$   $\boxed{2}$   $\boxed{\blacktriangleright}$   $\boxed{)}$   $\boxed{\text{a}^{\text{b}}}$   $\boxed{(-)}$   $\boxed{\text{ALPHA}}$   $\boxed{\text{N}}$   $\boxed{\blacktriangleright}$   $\boxed{\blacktriangleright}$   
 $\boxed{\text{ALPHA}}$   $\boxed{\text{I}}$   $\boxed{\div}$   $\boxed{1}$   $\boxed{2}$   $\boxed{\blacktriangleright}$   $\boxed{\blacktriangleright}$   $\boxed{)}$   $\boxed{\text{a}^{\text{b}}}$   $\boxed{(-)}$   $\boxed{1}$ .

This equation is referred to as the amortization formula, with a loan (L) with a fixed rate of interest (I).

4. Press  $\boxed{\text{ENTER}}$  to view the variable list. To find the monthly payment on a \$15,000 car loan made at 9% interest over four years (48 months), enter the values by pressing  $\boxed{\blacktriangledown}$   $\boxed{1}$   $\boxed{5}$   $\boxed{0}$   $\boxed{0}$   $\boxed{0}$   $\boxed{\text{ENTER}}$   $\boxed{\cdot}$   $\boxed{0}$   $\boxed{9}$   $\boxed{\text{ENTER}}$   $\boxed{4}$   $\boxed{8}$   $\boxed{\text{ENTER}}$ . Press  $\boxed{\blacktriangle}$   $\boxed{\blacktriangle}$   $\boxed{\blacktriangle}$  and notice the payment (P) is now highlighted by the cursor, press  $\boxed{2\text{ndF}}$   $\boxed{\text{EXE}}$  to solve for the payment.
5. Pressing  $\boxed{\text{CL}}$  will return you to the variable screen. You can now change or solve for any of the values. Save this formula by pressing  $\boxed{2\text{ndF}}$   $\boxed{\text{SOLVER}}$ , touching **C SAVE**, pressing  $\boxed{\text{ENTER}}$ , and entering the formula name. Give the formula the name AMORT by pressing  $\boxed{\text{A}}$   $\boxed{\text{M}}$   $\boxed{\text{O}}$   $\boxed{\text{R}}$   $\boxed{\text{T}}$   $\boxed{\text{ENTER}}$ .

## 3.2

**FORMULAS OR LITERAL EQUATIONS****Steps for solving an equation using to graphic method**

1. While viewing the variable list within the solver feature, press  $\boxed{\text{CL}}$   $\boxed{\text{CL}}$  to return to and clear the formula entry screen. Select the Graphic method for solving by pressing  $\boxed{2\text{ndF}}$   $\boxed{\text{SOLVER}}$ , touch **A METHOD**, and double touch **3 Graphic**.
2. Enter the formula  $V = \pi r^2 h$  by pressing  $\boxed{\text{ALPHA}}$   $\boxed{\text{V}}$   $\boxed{\text{ALPHA}}$   $\boxed{=}$   $\boxed{2\text{ndF}}$   $\boxed{\pi}$   $\boxed{\text{ALPHA}}$   $\boxed{\text{R}}$   $\boxed{a^b}$   $\boxed{2}$   $\boxed{\blacktriangleright}$   $\boxed{\text{ALPHA}}$   $\boxed{\text{H}}$ .
3. This equation is the formula for calculating the volume of a cylinder (V) in terms of the cylinder's height (H) and radius (R). Press  $\boxed{\text{ENTER}}$  to view the variable list.
4. To find the radius of a cylinder with a volume of 30 cubic inches, and a height of 10 inches, enter the values by pressing  $\boxed{3}$   $\boxed{0}$   $\boxed{\text{ENTER}}$   $\boxed{\blacktriangledown}$   $\boxed{1}$   $\boxed{0}$   $\boxed{\text{ENTER}}$ . Press  $\boxed{\blacktriangle}$  and notice the radius (R) is now highlighted by the cursor.
5. Press  $\boxed{2\text{ndF}}$   $\boxed{\text{EXE}}$  to solve for the radius. The graphic solver will prompt you for a variable range to solve within. Set the variable range to 0 and 2 by pressing  $\boxed{0}$   $\boxed{\text{ENTER}}$   $\boxed{2}$   $\boxed{\text{ENTER}}$ .
6. Press  $\boxed{2\text{ndF}}$   $\boxed{\text{EXE}}$  to solve.
7. Pressing  $\boxed{\text{CL}}$  will return you to the variable screen. You can now change or solve for any of the values. Save this formula by pressing  $\boxed{2\text{ndF}}$   $\boxed{\text{SOLVER}}$ , touching **C SAVE**, pressing  $\boxed{\text{ENTER}}$ , and entering the formula name. Give the formula the name VCYL by pressing  $\boxed{\text{V}}$   $\boxed{\text{C}}$   $\boxed{\text{Y}}$   $\boxed{\text{L}}$   $\boxed{\text{ENTER}}$ .

## 3.3

# FORMULAS OR LITERAL EQUATIONS

## Solving an equation using Newton's method

1. Press **2ndF** **SOLVER** to enter the solver feature. Press **CL** to clear the formula entry screen.
2. Select the Newton's method for solving by pressing **2ndF** **SOLVER**, touching **A METHOD**, and double touching **2 Newton**. Enter the formula  $A = \frac{1}{2} H(B + C)$  by pressing **ALPHA** **A** **ALPHA** **=** **1** **a/b** **2** **▶** **ALPHA** **H** **(** **ALPHA** **B** **+** **ALPHA** **C** **)**.
3. Press **ENTER** to view the variable list. To find the height of a trapezoid with an area of 25 in<sup>2</sup>, and bases of length 5 and 7 inches, enter the values by pressing **2** **5** **ENTER** **▼** **5** **ENTER** **7** **ENTER** **▲** **▲**. Notice the height (H) is now highlighted by the cursor.
4. Press **2ndF** **EXE** to solve for the height. Newton's method will prompt you for a guess or starting point. Enter a starting point of 1 by pressing **1** **ENTER**.
5. Press **2ndF** **EXE** to solve. A height of 4.1667 will appear on the screen.
6. Pressing **CL** will return you to the variable screen. You can now change or solve for any of the values. Save this formula by pressing **2ndF** **SOLVER**, touching **C SAVE**, pressing **ENTER**, and entering the formula name. Give the formula the name "ATRAP," by pressing **A** **T** **R** **A** **P** **ENTER**.
7. Find the height of a trapezoid with an area of 50 in<sup>2</sup> with bases of 8 and 10.

4.1

# GRAPHING POLYNOMIALS AND FINDING THE ROOTS

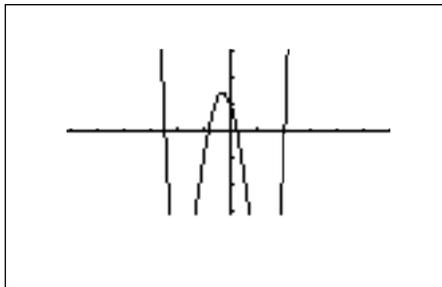
## Graphing a polynomial and zooming to find the roots

1. Turn the calculator on and press  $\boxed{Y=}$ . Y prompts will appear on the viewing window. Press  $\boxed{CL}$  to remove old Y expressions. Setup the calculator with rectangular coordinates and the equation editor mode by pressing  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press  $\boxed{CL}$  to exit the menu and return to the Y prompts.
2. To enter the polynomial  $y = x^3 - 3x^2 + x + 1$ , press  $\boxed{EZ}$ , double touch **5 X<sup>3</sup>**, touch **A Y=AX<sup>3</sup>+BX<sup>2</sup>+CX+D**, touch **1Y=AX<sup>3</sup> + BX + CX + D**, press  $\boxed{2ndF}$   $\boxed{SUB}$  and enter the coefficients by pressing  $\boxed{1}$   $\boxed{ENTER}$   $\boxed{(-)}$   $\boxed{3}$   $\boxed{ENTER}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{1}$   $\boxed{ENTER}$ .
3. Press  $\boxed{ZOOM}$ , touch  $\blacktriangledown$  on the screen, and touch **7 Dec** to establish the decimal viewing window and view the graph. Press  $\boxed{TRACE}$  to engage the trace feature. Press  $\blacktriangleleft$  to move the cursor near the left-hand root.
4. Set the zoom factors to 5 by pressing  $\boxed{ZOOM}$ , touch **B FACTOR**, press  $\boxed{ENTER}$   $\boxed{5}$   $\boxed{ENTER}$   $\boxed{5}$   $\boxed{ENTER}$ . Press  $\boxed{ZOOM}$ , touch **A ZOOM**, double touch **3 IN** to zoom in on the left-hand root. Press  $\boxed{TRACE}$  and move the tracer to approximate the root.
5. Press  $\boxed{ZOOM}$ , touch **H RCL**, double touch **2 PreWin** to return to the decimal viewing window. Press  $\boxed{TRACE}$  and move the tracer to find the middle root.
6. Press  $\blacktriangleright$  repeatedly to move the tracer near the right-hand root. Press  $\boxed{ZOOM}$ , touch **A ZOOM**, double touch **3 In** to zoom in. Press  $\boxed{TRACE}$  and move the tracer to approximate the root.

## 4.2

**GRAPHING POLYNOMIALS AND FINDING THE ROOTS****Graphing a polynomial and jumping to find the roots**

1. Press  $\boxed{Y=}$   $\boxed{CL}$  to return to and clear the Y1 prompt.
2. To enter the polynomial  $y = x^4 + x^3 - 5x^2 - 3x + 1$ , press  $\boxed{X/\theta/T/n}$   $\boxed{a^b}$   $\boxed{4}$   $\boxed{\blacktriangleright}$   $\boxed{+}$   $\boxed{X/\theta/T/n}$   $\boxed{a^b}$   $\boxed{3}$   $\boxed{\blacktriangleright}$   $\boxed{-}$   $\boxed{5}$   $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{-}$   $\boxed{3}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{1}$ .
3. View the graph in the decimal viewing window by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and touching **7 Dec**.



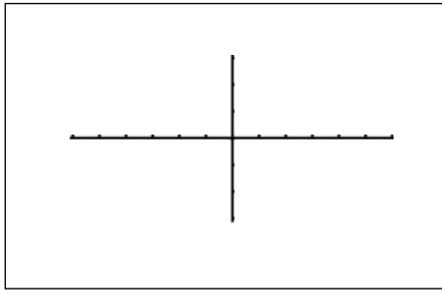
4. Press  $\boxed{TRACE}$  and move cursor near the right-hand root. Press  $\boxed{2ndF}$   $\boxed{CALC}$  to view the calculate menu.
5. Double touch **5 X\_Incpt** to find the left-hand root.
6. Press  $\boxed{2ndF}$   $\boxed{CALC}$ , double touch **5 X\_Incpt** to find the next root.
7. Press  $\boxed{2ndF}$   $\boxed{CALC}$ , double touch **5 X\_Incpt** to find the next root.
8. Press  $\boxed{2ndF}$   $\boxed{CALC}$ , double touch **5 X\_Incpt** to find the next root.

## 4.3

**GRAPHING POLYNOMIALS AND FINDING THE ROOTS****Graphing polynomials and zooming to find the roots**

Graph the following in a decimal viewing window and sketch the curve in the box provided ( $-6.3 < x < 6.3$ ,  $-3.1 < y < 3.1$ , scales of 1). Use the zoom feature to approximate the roots.

(1)  $y = x^3 - 2x^2 - 7x + 11$

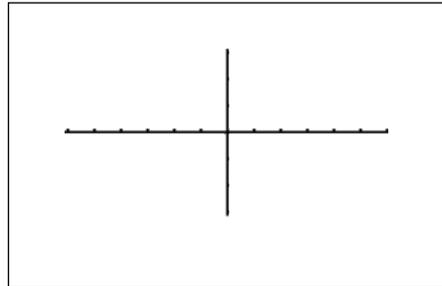


roots = \_\_\_\_\_

**Graphing polynomials and using the calculate feature to find the roots**

Graph the following in a decimal viewing window and sketch the curve in the box provided. Use the calculate feature to approximate the roots.

(2)  $y = x^3 + 5x^2 + 6x + 1$



roots = \_\_\_\_\_

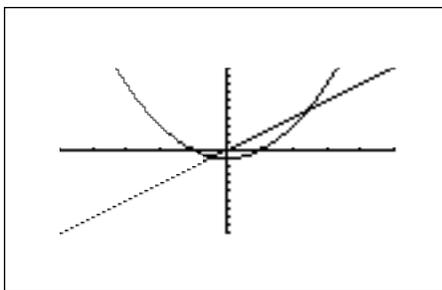
5.1

## SOLVING A SYSTEM OF EQUATIONS

### Graphing a system of equations and using the calculate feature to find the solutions

1. Turn the calculator on and press  $\boxed{Y=}$ .
2. Press  $\boxed{CL}$  to clear an old Y1 expression. Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear additional Y prompts.
3. Set the calculator to rectangular coordinates and in equation editor mode. To complete this setup, press  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, double touch **1 Rect**, touch **G EDITOR**, and double touch **1 Equation**. Press  $\boxed{CL}$  to exit the menu and return to the Y prompts.
4. To enter the system of equations:
 
$$y = x^2 - 1$$

$$y = 2x$$
 press  $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{-}$   $\boxed{1}$   $\boxed{ENTER}$   $\boxed{2}$   $\boxed{X/\theta/T/n}$ . View the graphs using the rapid window by pressing  $\boxed{WINDOW}$   $\boxed{EZ}$ , touch **5**, touch  $\blacktriangledown$  on the screen, double touch **7**  $-5 < X < 5$ , and double touch **4**  $-10 < Y < 10$ .



5. Press  $\boxed{2ndF}$   $\boxed{CALC}$  to access the calculate feature. Double touch **2 Intsct**. The left-hand intersection will appear on the screen.
6. Press  $\boxed{2ndF}$   $\boxed{CALC}$  to access the calculate feature again. Double touch **2 Intsct**. The right-hand intersection will appear on the screen.

# 5.2

## SOLVING A SYSTEM OF EQUATIONS

### Solving a system of linear equations using the tool feature

1. Press **2ndF** **TOOL** to access the tool menu.
2. Touch **B SYSTEM** and double touch **2 2** to view the entry screen for solving a linear-system of equations. Systems up to six variables and six equations can be solved.

3. To enter the system of equations:  $5x + y = 1$   
 $-3x + y = -5$

Press **5** **ENTER** **1** **ENTER** **1** **ENTER** **(-)** **3** **ENTER** **1**  
**ENTER** **(-)** **5** **ENTER** .

	a	b	c
1	5	1	1
2	-3	1	-5
			-5

4. Press **2ndF** **EXE** to solve the system.

```

aX+bY=c
X=.75
Y=-2.75
    
```

5.3

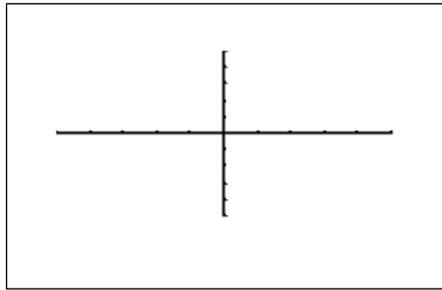
## SOLVING A SYSTEM OF EQUATIONS

### Activity 1 Graphing a system of equations and using the calculate feature to find the solutions

Graph the following system of equations and sketch the graphs in the box provided ( $-5 < x < 5$ ,  $-5 < y < 5$ , scales of 1). Use the calculate feature to approximate the solutions.

$$y = x^2 - 4x + 3$$

$$y = -x^2 + 2x + 2$$



solutions = \_\_\_\_\_

### Activity 2 Using the tool feature to find the solution to a system of linear equations

$$x + y = 1$$

$$-2x + y = -1$$

solutions = \_\_\_\_\_

6.1

# MATRIX SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS

1. Press **MATRIX** to access the matrix menu.
2. Touch **B EDIT** and double touch **1 mat A** to select matrix A.
3. Now, enter the size or dimension of the matrix. We will enter

the  $3 \times 3$  matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$$

Press **3** **ENTER** **3** **ENTER** to set the dimension of the matrix at three rows by three columns.

4. The calculator will now prompt you for the matrix. Enter the elements of the matrix by pressing **1** **ENTER** **2** **ENTER** **1** **ENTER** **2** **ENTER** **1** **ENTER** **(-)** **1** **ENTER** **1** **ENTER** **1** **ENTER** **(-)** **2** **ENTER**.
5. Press **2nd** **QUIT** to exit the display of matrix A.
6. Repeat the process to enter a  $3 \times 3$  matrix B =  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ .
7. Matrix multiplication can be performed if the number of columns of the first matrix is equal to the number of rows of the second matrix. In the matrix multiplication  $A \times B$ , the elements in the first row of A are multiplied to the corresponding elements in the first column of B. The sum of these multiplications is placed in the 1,1 (first row, first column) position of the resulting matrix. This process is repeated until each row of A has been multiplied to each column of B.
8. To multiply the matrices A and B together, press **MATRIX**, touch **A NAME**, double touch **1 mat A**, press  **$\times$**  **MATRIX**, double touch **2 mat B**, and press **ENTER**.

## 6.2

# MATRIX SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS

- The calculator will directly establish an identity matrix of a given size by pressing **MATRIX**, touching **C OPE**, double touching **05 identity**, and pressing **3** **ENTER**. To save the identity matrix in matrix C, press **STO** **MATRIX**, touch **A NAME**, touch **3 mat C**, and press **ENTER**. Confirm that the identity matrix is stored in matrix C by pressing **MATRIX**, touching **B EDIT**, and double touching **3 mat C**. Press **2ndF** **QUIT** to exit the matrix editor and press **CL** to clear the screen.
- Find the inverse of the square matrix A by pressing **MATRIX**, touching **A NAME**, double touching **1 mat A**, pressing **2ndF** **x<sup>-1</sup>** **ENTER**. Press **▶** to see more of the matrix.
- To solve the system of equations
 
$$\begin{aligned}x + 2y + z &= 8 \\2x + y - z &= 1 \\x + y - 2z &= -3\end{aligned}$$
 using matrices, use the matrix A entered previously as the coefficient matrix, and enter the constants on the right side of the equal sign into matrix B, where  $B = \begin{bmatrix} 8 \\ 1 \\ -3 \end{bmatrix}$ . Press **2ndF** **QUIT** to exit the display of the B matrix. The solution matrix X is found by multiplying  $A^{-1} B$ .
- This multiplication is derived from the equation  $AX = B$ ,  
 $A^{-1} A X = A^{-1} B$  (multiply both sides by  $A^{-1}$ )  
 $I X = A^{-1} B$  ( $A^{-1}A = I$ , identity matrix)  
 $X = A^{-1} B$  ( $I X = X$ )  
 Multiply  $A^{-1} B$  by pressing **MATRIX**, touching **A NAME**, double touching **1 mat A**, pressing **2ndF** **x<sup>-1</sup>** **×** **MATRIX** double touching **2 mat B**, and pressing **ENTER**. The solution matrix will appear.

6.3

# MATRIX SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS

**Activity 1** Solve the following system of linear equations.

$$x - 2y + 3z = 9$$

$$-x + 3y = -4$$

$$2x - 5y + 5z = 17$$

1. Enter the coefficient matrix A.
2. Enter the constant matrix B.
3. Find  $A^{-1}B$ .
4. The solution is  $(x, y, z) = ( \quad, \quad, \quad )$

**Activity 2** Solve the following system of linear equations using the tool feature.

$$x - 3z = -2$$

$$3x + y - 2z = 5$$

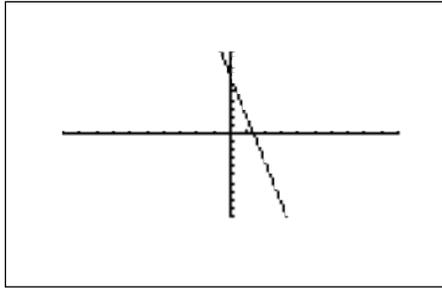
$$2x + 2y + z = 4$$

1. Press  $\boxed{2ndF}$   $\boxed{TOOL}$ , touch **B SYSTEM**, and double touch **3 3**.
2. Enter the coefficients and constants in the system solver.
3. Press  $\boxed{2ndF}$   $\boxed{EXE}$ .
4. The solution is  $(x, y, z) = ( \quad, \quad, \quad )$

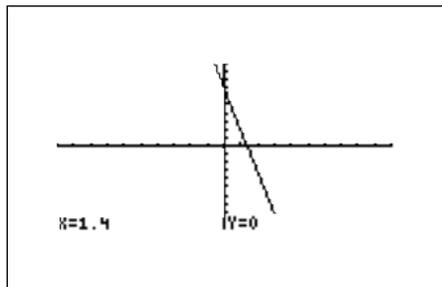
## 7.1

**INEQUALITIES**

- To solve  $3(4 - 2x) \geq 5 - x$ , rewrite it as  $3(4 - 2x) - 5 + x \geq 0$  and determine the values of  $x$  where the function  $y = 3(4 - 2x) - 5 + x$  is on or above the  $x$ -axis.
- To do this, press **Y=** **CL** and enter  $3(4 - 2x) - 5 + X$  in the Y1 location.
- Set the viewing window of the graph by pressing **ZOOM**, touch **A ZOOM**, and double touch **5 Default**. You should be able to clearly view the  $x$ -intercept.



- Locate the  $x$ -intercept at the point  $(1.4, 0)$  by pressing **TRACE** **2ndF** **CALC**, and double touch **5 X\_Incpt**.

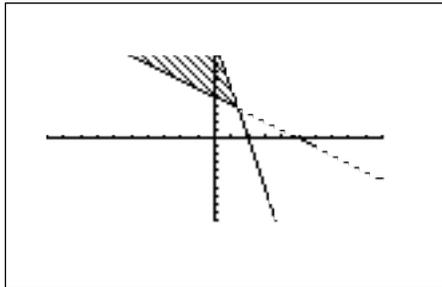


- Since the graph is above the  $x$ -axis to the left of the  $x$ -intercept, the solution to the inequality  $3(4 - 2x) - 5 + x \geq 0$  is all values of  $x$  such that  $x \leq 1.4$ .

## 7.2

**INEQUALITIES**

1. To solve the inequality  $3(4 - 2x) \geq 5 - x$ , press  $\boxed{Y=}$   $\boxed{CL}$ , enter  $3(4 - 2X)$  for  $Y1$  and  $5 - X$  for  $Y2$ .
2. Set the viewing window by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, and double touching **5 Default**.
3. Next, shade the set of points that make the inequality true by pressing  $\boxed{2ndF}$   $\boxed{DRAW}$ , touching **G SHADE**, and double touching **1 Set** to access the Set Shade screen. Since the inequality you are solving is  $Y1 \geq Y2$  the solution is where the graph of  $Y1$  is "on the top" and  $Y2$  is "on the bottom." Do this by touching **Y2** on the screen, pressing  $\boxed{\blacktriangleright}$ , and touching **Y1** on the screen. Press  $\boxed{GRAPH}$  to view the shaded region.

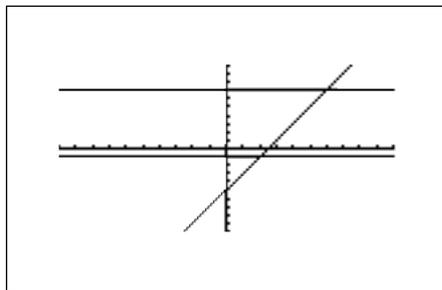


4. Press  $\boxed{TRACE}$   $\boxed{2ndF}$   $\boxed{CALC}$ , and double touch **2 Intset** to find where the graphs intersect.
5. Since the shaded region is to the left of  $x = 1.4$ , the solution to the inequality  $3(4 - 2x) \geq 5 - x$  is all values of  $x$  such that  $x \leq 1.4$ .

## 7.3

**DOUBLE INEQUALITIES**

1. The inequality  $-1 \leq 2x - 5 \leq 7$  is commonly referred to as a “double” inequality.
2. Turn off the shading by pressing **2ndF** **DRAW**, touching **G SHADE**, and double touching **2 INITIAL**.
3. Clear any previously entered functions by pressing **Y=** **CL**.
4. Enter  $Y1 = -1$ ,  $Y2 = 2X - 5$ , and  $Y3 = 7$ .
5. Press **ZOOM**, touch **A ZOOM**, and double touch **5 Default** to view the line  $y = 2x - 5$  between the lines  $y = -1$  and  $y = 7$ .



6. Press **TRACE** **2ndF** **CALC**, double touch **2 Intsct** to find the point of intersection of the lines  $y = 2x - 5$  and  $y = -1$  at  $(2, -1)$ . Press **▲** to move the tracer to the  $y = 7$  line. Press **2ndF** **CALC** and double touch **2 Intsct** to find  $y = 2x - 5$  and  $y = 7$  at  $(6, 7)$ .
7. The solution to the “double” inequality  $-1 \leq 2x - 5 \leq 7$  consists of all values of  $x$  in between, and including, 2 and 6 (i.e.,  $x \geq 2$  and  $x \leq 6$ ). The solution is  $2 \leq x \leq 6$ .

## 7.4

# SYSTEMS OF INEQUALITIES

1. The solution region of a system of inequalities consists of all points  $(a, b)$  such that when  $x = a$  and  $y = b$ , all inequalities in the system are true.

2. For example, to graph the solution region for the system

$$2x + y \geq 1$$

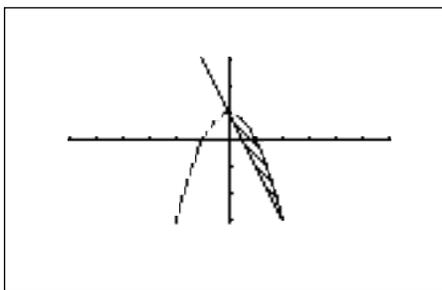
$$x^2 + y \leq 1,$$

first rewrite each inequality in the system so that the left-hand-side is  $y$ .

$$y \geq 1 - 2x$$

$$y \leq 1 - x^2$$

3. Press  $\boxed{Y=}$   $\boxed{CL}$  and enter  $1 - 2X$  in  $Y1$  and  $1 - X^2$  in  $Y2$ .
4. Next, shade the points that have a  $y$ -value less than  $Y2$  and greater than  $Y1$ . Do this by pressing  $\boxed{2ndF}$   $\boxed{DRAW}$ , touching **G SHADE**, double touching **1 Set**, touch **Y1** on the screen, press  $\blacktriangleright$ , and touch **Y2** on the screen.
5. Graph the system by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and touching **7 Dec**.



7.5

## INEQUALITIES AND SYSTEMS OF INEQUALITIES

1. Use the first method to solve the following inequalities:

- a.  $\frac{1}{2}x + 4 \geq 3x$   
 b.  $5 - x^2 + 2x < x + 3$

2. Use the second method to solve the following inequalities:

- a.  $\frac{3 - 2x}{5} \leq \frac{1}{2}x - 4$   
 b.  $x^3 > (x - 2)^2$

3. Find the values of  $x$  for which:

- a.  $4x - 5 > 1 - 2x^2$   
 b.  $2(x + 2)^2 - 3 < 1 - x^2$   
 c.  $\sqrt{4 + x} < x$   
 d.  $\ln x + \frac{3x}{5} \geq e^x$

4. a. Solve the inequality  $-3 \leq 2 - x \leq 4$   
 b. Solve the inequality  $-8 < \frac{6x - 2}{5} \leq -2$

5. a. Graph the solution set of the system of inequalities:

$$x + 2y \leq 1$$

$$x^2 + y \geq 4$$

- b. Which of the following points are within the solution region?  $(-1.6, 1.8)$ ,  $(-2, -5)$ ,  $(2.8, -1.4)$ ,  $(8, 4)$

6. Graph the solution set of the system of inequalities:

$$2y + 3x \geq 8$$

$$2y + 3x \leq 12$$

$$x \geq 0$$

$$y \geq 1$$

## 8.1

# ABSOLUTE VALUE FUNCTIONS

1. Evaluate  $|-2(5 - 1)|$  by pressing  $\left[ \begin{smallmatrix} + & = \\ \times & \div \end{smallmatrix} \right]$  to access the home or computation screen. Enter the absolute value symbol by pressing  $\left[ \text{MATH} \right]$ , touching **B NUM**, double touching **1 abs(**. You should see  $| \quad |$  on the screen. Enter  $-2(5-1)$  inside the absolute value symbols and evaluate by pressing  $\left[ (-) \right] \left[ 2 \right] \left[ ( \right] \left[ 5 \right] \left[ - \right] \left[ 1 \right] \left[ ) \right] \left[ \text{ENTER} \right]$ .

$|-2(5-1)|$ 
8

2. Graph  $y = |x|$  by pressing  $\left[ \text{Y=} \right] \left[ \text{CL} \right]$  to clear the Y1 prompt. Press  $\left[ \text{ENTER} \right]$   $\left[ \text{CL} \right]$  to clear the remaining prompts if necessary. Enter  $|x|$  in Y1 by pressing  $\left[ \text{MATH} \right]$  touching **B NUM**, double touching **1 abs(** and pressing  $\left[ \text{X}/\theta/\text{T}/n \right]$ . Press  $\left[ \text{ZOOM} \right]$ , touch **A ZOOM**, and double touch **5 Default** to draw the graph.
3. Graph  $y = |x - 1|$  by pressing  $\left[ \text{Y=} \right] \left[ \text{CL} \right]$  to clear the Y1 prompt. Press  $\left[ \text{EZ} \right]$ , double touch **8 |x|**, touch **A Y = A |X - H| + K**, touch **1 Y = A |X - H| + K**, and press  $\left[ 2\text{ndF} \right] \left[ \text{SUB} \right] \left[ 1 \right] \left[ \text{ENTER} \right] \left[ 1 \right] \left[ \text{ENTER} \right] \left[ 0 \right] \left[ \text{ENTER} \right] \left[ \text{ZOOM} \right] \left[ 5 \right]$ .
4. Now, graph  $y = |x| - 1$  by pressing  $\left[ \text{Y=} \right] \left[ 2\text{ndF} \right] \left[ \text{SUB} \right] \left[ 1 \right] \left[ \text{ENTER} \right] \left[ 0 \right] \left[ \text{ENTER} \right] \left[ (-) \right] \left[ 1 \right] \left[ \text{ENTER} \right] \left[ \text{GRAPH} \right]$ .

# 8.2

## ABSOLUTE VALUE FUNCTIONS

### Activity 1 Direct evaluation of absolute values

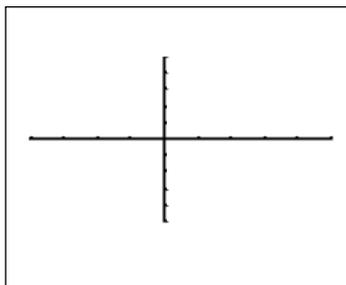
1.  $|-5| =$  \_\_\_\_\_      2.  $|4(2 - 6)| =$  \_\_\_\_\_      3.  $-4|-2| =$  \_\_\_\_\_

### Activity 2 Graphing an absolute value function

Sketch the graph of each function in the box below the problem.

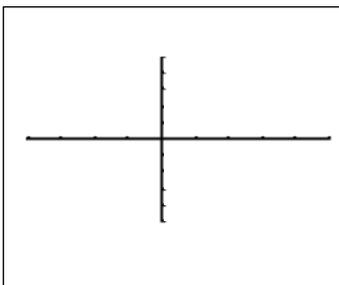
List the domain and range of each function in the blanks below the box.

1.  $y = |x + 2|$



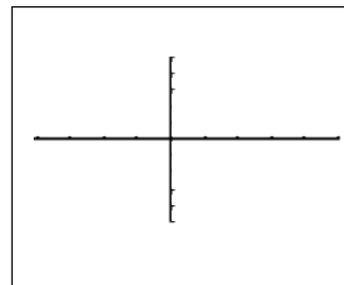
\_\_\_\_\_  
\_\_\_\_\_

2.  $y = |x| + 2$



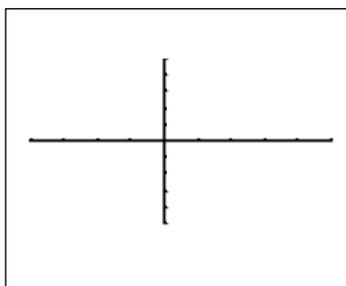
\_\_\_\_\_  
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3.  $y = -2|x|$



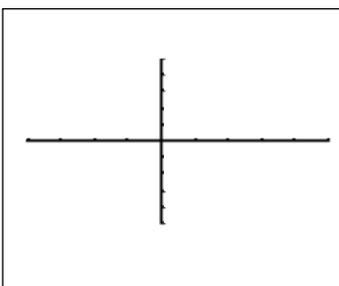
\_\_\_\_\_  
\_\_\_\_\_

4.  $y = -2|x - 1|$



\_\_\_\_\_  
\_\_\_\_\_

5.  $y = -2|x| - 1$

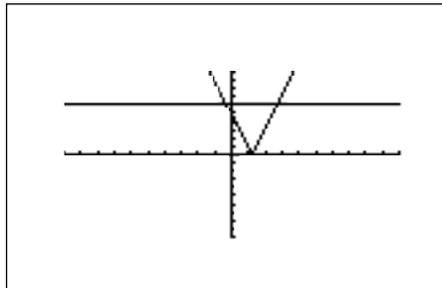


\_\_\_\_\_  
\_\_\_\_\_

## 8.3

**ABSOLUTE VALUE EQUATIONS**

1. Solve  $|5 - 4x| = 6$  by first pressing **Y=** **CL** to clear the Y1 prompt.
2. Enter  $|5 - 4x|$  in Y1 with the keystrokes **MATH**, touch **B NUM**, double touch **1 abs(** and press **5** **-** **4** **X/θ/T/n**.
3. Next, press **ENTER** **CL** **6** to enter 6 in Y2.
4. Press **ZOOM**, touch **A ZOOM**, double touch **5 Default** to view the two points of intersection of the absolute value graph and the horizontal line  $y = 6$ .

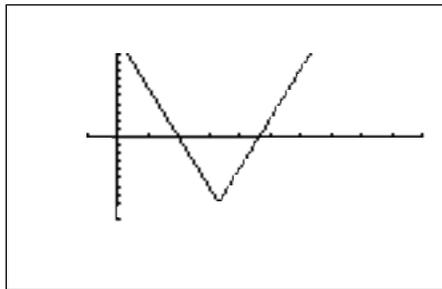


5. Press **TRACE** **2ndF** **CALC** and double touch **2 Intsct** to find one point of intersection of the two graphs. Press **2ndF** **CALC** and double touch **2 Intsct** again to find the other point of intersection.
6. The solution to the equation  $|5 - 4x| = 6$  consists of the two values  $-0.25$  and  $2.75$ .

## 8.4

**ABSOLUTE VALUE INEQUALITIES**

- To solve  $\left| \frac{20 - (6x)}{5} \right| < 8$  using the first method, rewrite the inequality so that the right-hand side of the inequality is zero:  $\left| \frac{20 - (6x)}{5} \right| - 8 < 0$ .
- Press  $\boxed{Y=}$  and clear all the Y prompts.
- Enter the left-hand side of the inequality  $\left| \frac{20 - (6x)}{5} \right| - 8$  in Y1 by pressing  $\boxed{\text{MATH}}$ , touching **B NUM**, double touching **1 abs(** (pressing  $\boxed{2}$   $\boxed{0}$   $\boxed{-}$   $\boxed{\text{a/b}}$   $\boxed{6}$   $\boxed{\text{X}/\theta/\text{T}/n}$   $\boxed{\blacktriangleright}$   $\boxed{5}$   $\boxed{\blacktriangleright}$   $\boxed{\blacktriangleright}$   $\boxed{-}$   $\boxed{8}$ ).
- Choose a viewing window by pressing  $\boxed{\text{WINDOW}}$   $\boxed{\text{EZ}}$ , double touching **6** on the screen, double touching **4**  $\cdot 5 < X < 50$ , and double touching **4**  $\cdot 10 < Y < 10$ . The  $x$ -intercepts of the graph are clearly visible.

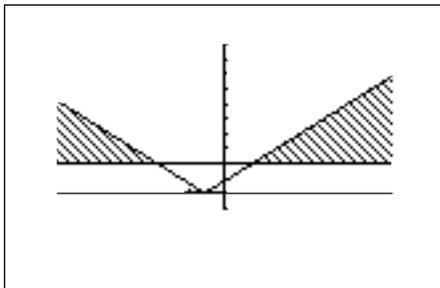


- Press  $\boxed{\text{TRACE}}$  and move right of the right intercept. Press  $\boxed{2\text{ndF}}$   $\boxed{\text{CALC}}$  and double touch **5 X\_Incpt** to find the first  $x$ -intercept. Press  $\boxed{2\text{ndF}}$   $\boxed{\text{CALC}}$  and double touch **5 X\_Incpt** to find the second  $x$ -intercept.
- Since the graph is below the  $x$ -axis for  $x$  in between the two  $x$ -intercepts, the solution is  $10 < x < 23.333$ .

## 8.5

**ABSOLUTE INEQUALITIES**

- To use the second method to solve the inequality  $|3.5x + 4| \geq 10$ , first press **Y=** **CL** to clear the Y1 prompt. Press **ENTER** **CL** to clear additional prompts.
- Enter the function  $|3.5x + 4|$  in Y1 by pressing **MATH**, touching **B NUM**, double touching **1 abs(** (and pressing **3** **.** **5** **X/θ/T/n** **+** **4** **ENTER**). Enter 10 in Y2 by pressing **1** **0**.
- Access the Set Shade screen by pressing **2ndF** **DRAW**, touching **G SHADE**, and double touching **1 SET**. Since Y2 is the function "on the bottom," touch **Y2** on the screen, and since Y1 is the function "on the top," press **▶** and touch **Y1** on the screen.
- Choose a viewing window by pressing **WINDOW** **EZ**, double touching **2** on the screen, double touching **5** **-10<X<10**, and double touching **3** **-5<Y<50**.



- Press **TRACE** **2ndF** **CALC**, double touch **2 Intsct** to locate a point of intersection. Repeat to find the other. The intersections are  $(-4, 10)$  and  $(1.714, 10)$ . The solution of the inequality  $|3.5x + 4| > 10$  is all values of  $x$  such that  $x < -4$  or  $x \geq 1.714$ .

## 8.6

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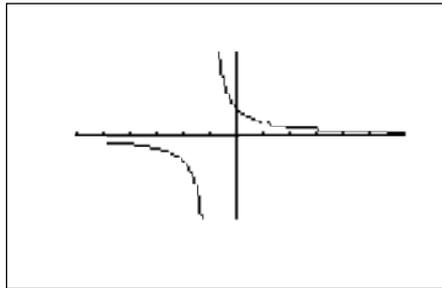
**ABSOLUTE VALUE FUNCTIONS, EQUATIONS,  
AND INEQUALITIES**

1. Find the values of  $x$  for which  $|3 - 8x| = 7$ .
2. Solve the equation  $\frac{5}{2} = |3\left(\frac{x}{2}\right) - 4|$ .
3. Solve the inequality  $|2x + 3| \leq 9$ .
4. For which values of  $x$  is  $|5x - 7| + 3 \geq 1$ ?
5. Solve:  $\left|\frac{(3x)}{2} - \frac{1}{5}\right| > 12$
6. Solve:  $|x^2 - 3x - 4| < 10$
7. Find the values of  $x$  for which  $|x + 2| \geq |3x - 1|$ .

## 9.1

**RATIONAL FUNCTIONS**

- Graph the rational function  $f(x) = \frac{(x-1)}{(x^2-1)}$  by first pressing **Y=** **CL** to clear the Y1 prompt. Press **ENTER** **CL** to clear additional Y prompts.
- Enter  $Y1 = \frac{(x-1)}{(x^2-1)}$  by pressing **a/b** **X/θ/T/n** **-** **1** **▶** **X/θ/T/n** **x<sup>2</sup>** **-** **1**.
- Graph in the decimal window by pressing **ZOOM**, touching **A ZOOM**, touching **▼** on the screen, and double touching **7 Dec**.

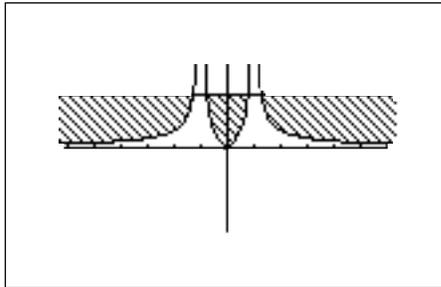


- The function consists of two branches separated by the vertical asymptote (the line  $x = -1$ ). Look closely at the graph and observe the "hole" in it at  $x = 1$ . Press **TRACE** and investigate the "hole." Note that the hole will be seen only when the window is a "friendly" or decimal window.
- Notice that there are no  $x$ -intercepts for the graph. The  $y$ -intercept can easily be found at  $x = 0, y = 1$  by pressing **TRACE** **2ndF** **CALC** and double touching **6 Y\_Incpt**.
- Observe that the line  $y = 0$  is very likely a horizontal asymptote of  $f(x)$ .

## 9.2

## RATIONAL INEQUALITIES

- Solve the inequality  $\left| \frac{x}{1-x^2} \right| \leq 2$  using the intersection method with shading, by first pressing  $\boxed{Y=}$   $\boxed{CL}$  to clear the Y1 prompt.
- Enter the left-hand side of the inequality in Y1 by pressing  $\boxed{\text{MATH}}$ , touching **B NUM**, double touching **1 abs**( pressing  $\boxed{a/b}$   $\boxed{X/\theta/T/n}$   $\boxed{\blacktriangleright}$   $\boxed{1}$   $\boxed{-}$   $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{\text{ENTER}}$  . Enter 2 in Y2 by pressing  $\boxed{2}$   $\boxed{\text{ENTER}}$  .
- Press  $\boxed{2\text{ndF}}$   $\boxed{\text{DRAW}}$ , touch **G SHADE**, double touch **1 SET**, touch **Y1** on the screen, press  $\boxed{\blacktriangleright}$ , and touch **Y2** on the screen.
- View the graph by pressing  $\boxed{\text{ZOOM}}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.



- Press  $\boxed{\text{TRACE}}$   $\boxed{2\text{ndF}}$   $\boxed{\text{CALC}}$  and double touch **2 Intsct**, repeatedly, to locate the  $x$ -values of the points of intersection,  $x = -1.281$ ,  $-0.781$ ,  $0.781$ , and  $1.281$ . The solution is all values of  $x$  such that  $x \leq -1.281$  or  $-0.781 \leq x \leq 0.781$  or  $x \geq 1.281$ . Turn off the shading by pressing  $\boxed{2\text{ndF}}$   $\boxed{\text{DRAW}}$ , touching **G SHADE**, and double touching **2 INITIAL**.

## 9.3

## RATIONAL FUNCTIONS AND INEQUALITIES

**Activity 1** Solve the rational inequality  $\frac{x^2}{x^2 - 4} \geq 2$

1. Enter  $\frac{x^2}{x^2 - 4}$  in Y1. Enter 2 in Y2.
2. Set up the shading.
3. View the graph in a decimal window.
4. Find the intersections of the rational function and the line  $y = 2$ .
5. Give the solution to the inequality by determining the values of  $x$  corresponding to the shaded regions on the graph.
6. Turn off the shading.

**Activity 2** Graph the rational function  $y = \frac{x - 3}{x^2 - 5x + 6}$

1. Enter  $\frac{(x - 3)}{(x^2 - 5x + 6)}$  in Y1.
2. Factor the denominator of  $y = \frac{(x - 3)}{(x^2 - 5x + 6)}$ . How many vertical asymptotes and how many branches do you expect to see when the graph of this rational function is drawn?
3. View the graph in a decimal window.
4. What is the domain of this rational function? What do you see in the graph to support your answer?

10.1

## CONIC SECTIONS

### Steps for graphing a parabola in rectangular mode

1. Graph the parabola  $x = y^2 - 2$  by first rewriting as  $y = \pm\sqrt{(x+2)}$ .
2. Press  $\boxed{Y=}$   $\boxed{CL}$  to clear the Y1 prompt. Press  $\boxed{ENTER}$   $\boxed{CL}$  to clear additional prompts. Enter  $\sqrt{(x+2)}$  in Y1 with the keystrokes  $\boxed{2ndF}$   $\boxed{\sqrt{\phantom{x}}}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{2}$ .
3. Press  $\boxed{ENTER}$  and enter  $-Y1 = -\sqrt{(x+2)}$  in Y2 with the key strokes  $\boxed{(-)}$   $\boxed{VARS}$ , touch **A EQVARS**, press  $\boxed{ENTER}$ , and double touch **1 Y1**.
4. View the graph by pressing  $\boxed{ZOOM}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.

### Steps for graphing a parabola in parametric mode

1. Change to parametric mode by pressing  $\boxed{2ndF}$   $\boxed{SET UP}$ , touch **E COORD**, and double touch **2 Param**. Press  $\boxed{2ndF}$   $\boxed{QUIT}$  to exit the set-up screen.
2. Press  $\boxed{Y=}$   $\boxed{CL}$  to clear the X1T prompt. To rewrite  $x = y^2 - 2$  in parametric form, simply let  $y = T$  and substitute in  $x = y^2 - 2$  to obtain  $x = T^2 - 2$ . Enter  $X1T = T^2 - 2$  in the calculator by pressing  $\boxed{X/\theta/T/n}$   $\boxed{x^2}$   $\boxed{-}$   $\boxed{2}$   $\boxed{ENTER}$ . Enter  $Y1T = T$  by pressing  $\boxed{X/\theta/T/n}$ .
3. View the graph by pressing  $\boxed{ZOOM}$ , touch **A ZOOM**, touch  $\blacktriangledown$  on the screen, and double touch **7 Dec**.
4. Notice that only a half of the parabola is drawn. To see the rest of the parabola, press  $\boxed{WINDOW}$  and adjust the Tmin. Set Tmin to  $-6$  by pressing  $\boxed{(-)}$   $\boxed{6}$   $\boxed{ENTER}$ . Press  $\boxed{GRAPH}$  to view the complete parabola.

10.2

## CONIC SECTIONS

### Steps for graphing a circle in rectangular mode

1. To graph the circle  $x^2 + y^2 = 4$ , first return to the rectangular graphing mode by pressing  $\boxed{2\text{ndF}} \boxed{\text{SET UP}}$ , touching **E COORD**, double touching **1 Rect**, and pressing  $\boxed{2\text{ndF}} \boxed{\text{QUIT}}$ .
2. Press  $\boxed{Y=}$   $\boxed{\text{CL}}$  to clear the Y1 prompt. Enter  $Y1 = \sqrt{4 - X^2}$  by pressing  $\boxed{2\text{ndF}} \boxed{\sqrt{\quad}} \boxed{4} \boxed{-} \boxed{X/\theta/T/n} \boxed{x^2} \boxed{\text{ENTER}}$  and then enter  $Y2 = \sqrt{4 - X^2}$  by pressing  $\boxed{\text{CL}}$   $\boxed{(-)}$   $\boxed{\text{VARS}}$ , touch **A EQVARS**, press  $\boxed{\text{ENTER}}$ , touch **A XY**, and double touch **1 Y1**. View the graph by pressing  $\boxed{\text{ZOOM}}$ , touching **A ZOOM**, touching  $\blacktriangledown$  on the screen, and double touching **7 Dec**.

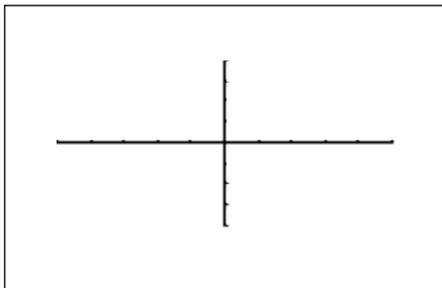
### Steps for graphing a circle not in standard form

1. First solve the equation for  $y$  by completing the square on the  $y$ -term and solving for  $y$ . For example, draw the graph of the circle  $x^2 - 2x + y^2 + 4y = 2$  by rewriting as  $y = \pm \sqrt{(6 - x^2 + 2x)} - 2$ .
2. Enter  $Y1 = \sqrt{(6 - x^2 + 2x)}$ ,  $Y2 = Y1 - 2$  and  $Y3 = -Y1 - 2$ . "Turn off" Y1 so that it will not graph by pressing  $\blacktriangle$  to move the blinking cursor to the Y1 expression. Press  $\blacktriangleleft$  to position the blinking cursor over the = sign, and press  $\boxed{\text{ENTER}}$  so that "=" is no longer darkened.
3. To view the graph, press  $\boxed{\text{ZOOM}}$ , touch  $\blacktriangledown$  on the screen, and double touch **7 Dec**.
4. Adjust the screen to see the bottom part of the circle. Do this using the rapid zoom feature by pressing  $\boxed{\text{EZ}}$ , double touching  $\blacktriangledown$  on the bottom of the screen twice, and double touching  $\blacktriangledown$  on the top of the screen twice. Press  $\boxed{\text{GRAPH}}$  to view the graph in the new window.

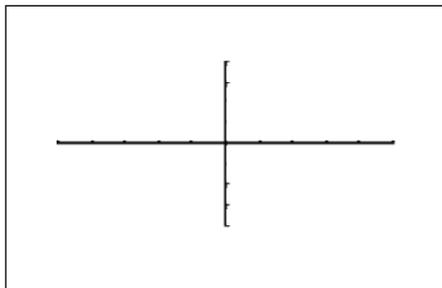
10.3

## CONIC SECTIONS

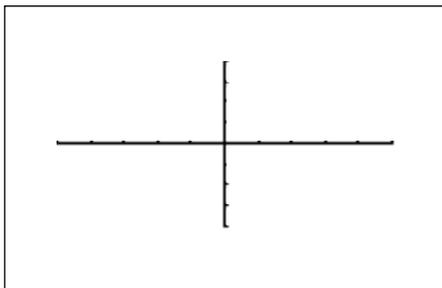
1. Graph the circle  $(x + 3)^2 + y^2 = 4$ .



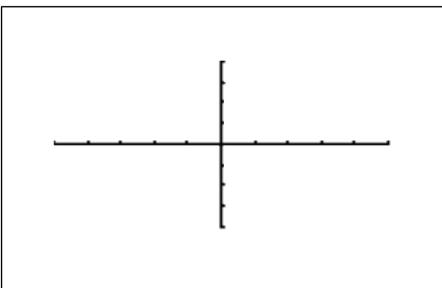
2. Graph the ellipse  $3(x - 3)^2 + (y + 2)^2 = 3$



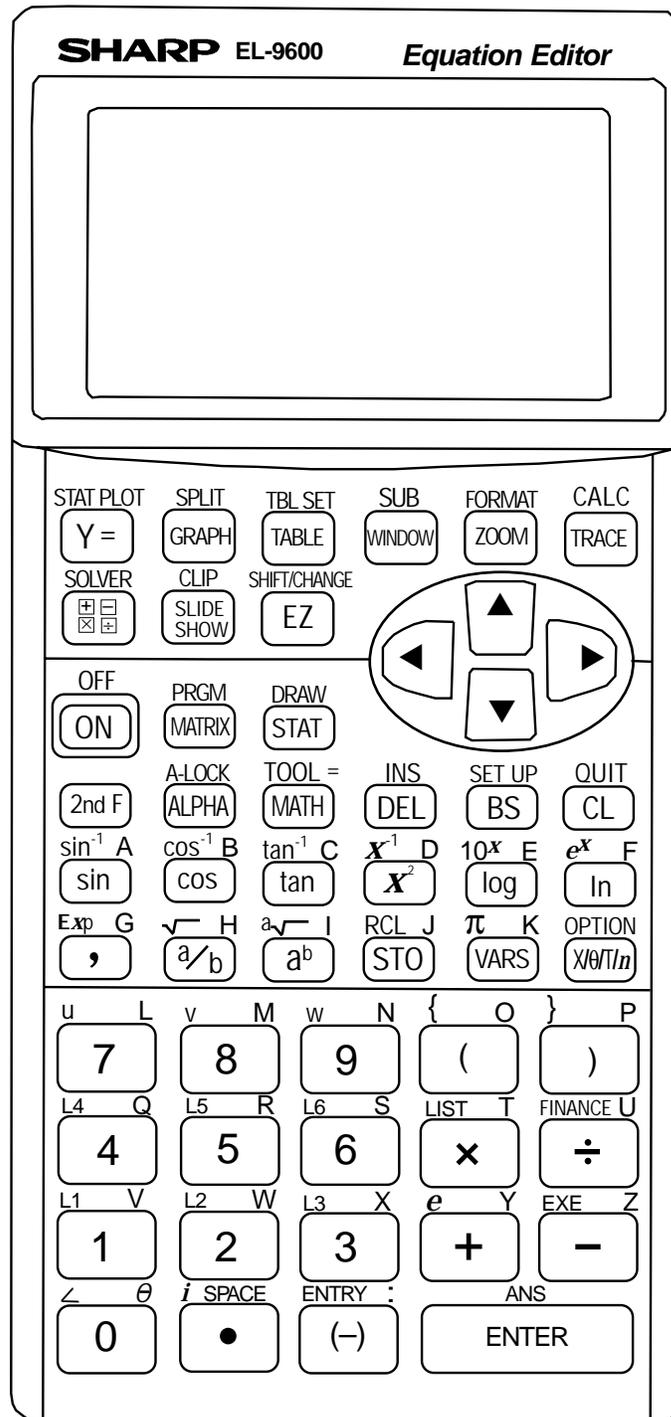
3. Graph the parabola  $x - y^2 = 4 - 2y$ .



4. Graph the hyperbola  $x^2 + 2x - y^2 - 6y + 3 = 0$ .



# KEYPAD FOR THE SHARP EL-9600 GRAPHING CALCULATOR



## SOLUTIONS TO THE ACTIVITIES

### 1. LINEAR EQUATIONS

#### BLACKLINE MASTER 1.3

5. moved up two
7. moved down two

#### BLACKLINE MASTER 1.4

2. parallel lines
5. perpendicular lines
7. neither, opposite angles are equal

### 2. QUADRATIC EQUATIONS

#### BLACKLINE MASTER 2.3

1. moved up one
2. opening
3. reflection
4. moved right three
5. closing, moved left one
6. reflection, opening, moved down three
7. closing, moved right one and up two
8. reflection, opening, moved left two and down one

### 3. FORMULAS OR

#### LITERAL EQUATIONS

#### ADDITIONAL PROBLEMS

1. \$313.36
2. \$15,150.60
3. 2.06

#### BLACKLINE MASTER 3.3

7. 5.56

### 4. GRAPHING

#### POLYNOMIALS AND FINDING THE ROOTS

#### ADDITIONAL PROBLEMS

1. -1.8, -.44, 1.3
2. -2.08, -.84, .84, 2.08
3. -1.4, -.75, .41, .79, 2.95

#### BLACKLINE MASTER 4.2

5. -2.47
6. -.82
7. .24
8. 2.05

#### BLACKLINE MASTER 4.3

1. -2.52, 1.4, 3.12
2. -3.25, -1.55, -2.0

### 5. SOLVING A SYSTEM OF EQUATIONS

#### ADDITIONAL PROBLEMS

1. (-1.62, -.62), (.62, 1.62)
2. (-2.45, -2.22), (-.45, -1.23), (.90, -.55)

#### BLACKLINE MASTER 5.1

5. (-.42, -.84)
6. (2.42, 4.86)

#### BLACKLINE MASTER 5.3

- Activity 1 (.18, 2.33), (2.82, -.31)
- Activity 2 (-1.62, -1.62), (.62, .62)

### 6. MATRIX SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS

#### ADDITIONAL PROBLEMS

1. (1, 0, 1)
2. (1, 0, 2)

#### BLACKLINE MASTER 6.2

4. (1, 2, 3)

#### BLACKLINE MASTER 6.3

- Activity 1 (1, -1, 2)
- Activity 2 (4, -3, 2)

### 7. INEQUALITIES AND SYSTEMS OF INEQUALITIES

#### ADDITIONAL PROBLEMS

1. All values of  $x$  such that  $x \leq 2$
2. All values of  $x$  such that  $x \geq 9.5$
3. a. All values of  $x$  between but not including -2 and -1 or all values greater than 3  
b. All real values of  $x$   
c. All values of  $x$  such that  $x > 0$   
d. All values of  $x$  such that  $-.5 < x < 1$
4. All values of  $x$  such that  $-2.4 \leq x \leq -1.6$
5. b. (.6, .5) is a solution; (3.5, 0) is not a solution
6. b. (-2, 0) is not a solution; (-1, 2.5) is a solution

#### BLACKLINE MASTER 7.5

1. a. The graph of  $Y1=(1/2)X + 4 - 3X$  is above the  $x$ -axis for  $x < 1.6$

- b. The graph of  $Y1=2 + X - X^2$  is below the  $x$ -axis for  $x < -1$  or  $x > 2$
2. a. The graph of  $Y1=(3-2x)/5$  is below the graph of  $Y2=(1/2)X-4$  for  $x > 5.1$   
b. The graph of  $Y1=x^3$  is above the graph of  $Y2=(x-2)^2$  for  $x > 1$
3. a. All values of  $x$  such that either  $x < -3$  or  $x > 1$   
b. All  $x$  such that  $x$  is between, but does not include, -2 and -.667  
c. All values of  $x$  such that  $x > 2.562$   
d. No real values of  $x$
4. a. All values of  $x$  such that  $-2 \leq x \leq 5$   
b. All values of  $x$  such that  $-6.333 \leq x \leq -1.333$
5. b. Points in the solution region are (2.8, -1.4) and (-8, 4). Points outside the solution region are (-1.6, 1.8) and (-2, -5). (Note: Range settings of -9.4  $< x < 9.4$ ,  $-6.2 < y < 6.2$  allow you to locate the points with the free-moving cursor.)
6.  $0 \leq x \leq 4.7$ ,  $1 \leq y \leq 6.2$

### 8. ABSOLUTE VALUE FUNCTIONS, EQUATIONS, AND INEQUALITIES

#### ADDITIONAL PROBLEMS

1.  $|-2 + 7| = 5$ ;  $|-2| + |7| = 9$ ; No; When  $x \geq 0$  and  $y \geq 0$  or  $x \leq 0$  and  $y \leq 0$ .
2. Both sides equal .75. The statement is true for all  $x$  and  $y \neq 0$ .
3.  $x = -2.5$ ,  $x = 17.5$
4. All values of  $x$  such that  $-2 \leq x \leq -5$
5. All values of  $x$  such that  $x \leq -.8$  or  $x \geq 1.6$

#### BLACKLINE MASTER 8.2

#### ACTIVITY 1

1. 5
2. 16
3. -8

#### ACTIVITY 2

1. Domain: all real numbers  
Range:  $y \geq 0$
2. Domain: all real numbers  
Range:  $y \geq 2$

3. Domain: all real numbers  
Range:  $y \leq 0$
4. Domain: all real numbers  
Range:  $y \leq 0$
5. Domain: all real numbers  
Range:  $y \leq -1$

#### BLACKLINE MASTER 8.6

1.  $x = -0.5$ ,  $x = 1.25$
2.  $x = 1$ ,  $x = 4.333$
3.  $-6 \leq x \leq 3$
4. all real numbers of  $x$
5.  $x < -8.133$  or  $x > 7.867$
6.  $-2.531 < x < 5.531$
7.  $-.25 \leq x \leq 1.5$

### 9. RATIONAL FUNCTIONS

#### ADDITIONAL PROBLEMS

1. b. all real numbers except  $x = 0$   
c.  $x = 0$   
d.  $x$ -intercept at  $x = -3$ , no  $y$ -intercept  
e.  $y = 1$
2. b. all real numbers except  $x = -2, -1, 1$   
c.  $x = -2$ ,  $x = 1$   
d.  $x$ -intercept at  $x = 0$ ,  $y$ -intercept at  $y = 0$   
e.  $y = 0$
3.  $x \leq -2.591$  or  $x \geq -1.009$
4. all values of  $x$  such that  $x > -2$  except  $x = 1$

#### BLACKLINE MASTER 9.3

#### ACTIVITY 1

4.  $x = -2.828$ ,  $2.828$
5.  $-2.828 < x < 2$  or  $2 < x < 2.828$

#### ACTIVITY 2

2. 1 vertical asymptote and 2 branches
4. domain: all values of  $x$  such that  $x \neq 2$ ,  $x \neq 3$  a hole in the graph at  $x = 3$

### 10. CONIC SECTIONS

#### ADDITIONAL PROBLEMS

1. parabola opening right, vertex at (0, 4)
2. parabola opening left, vertex at (0, -1)
3. ellipse centered at the origin
4. parabola opening left
5. circle
6. hyperbola, opening up and down

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